LAST PUSH

MATHS IS MATHS

PAPER 2

GRADE 12

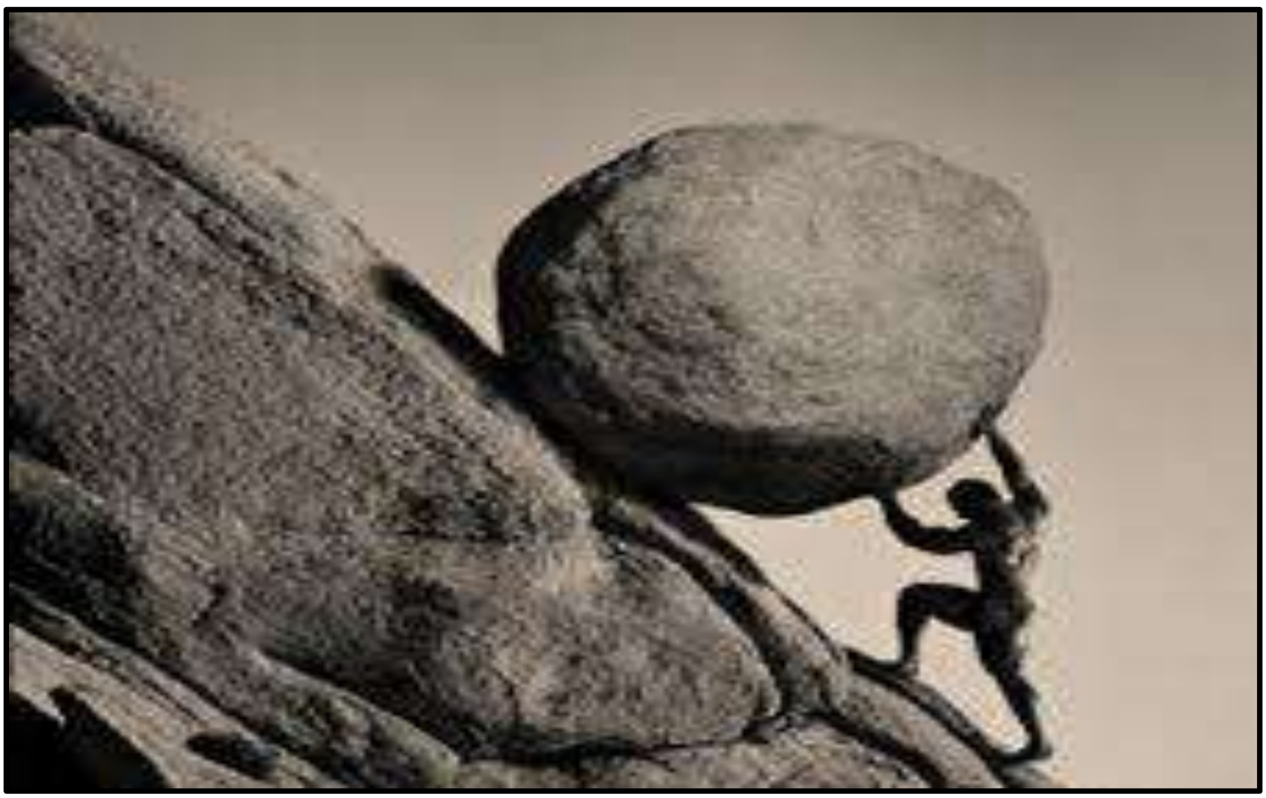

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TABLE OF CONTENTS

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STATISTICS

- Five number summary
- Box and whisker
- Cumulative frequency
- Ogive
- Scatter plot

QUESTION 1

The data in the table below represents the score in percentage of 12 Mathematics learners in their Grade 12 trial examination and their corresponding final examination.

Trial Exam	76	64	90	68	70	79	52	64	61	71	84	70
Final Exam	82	69	94	75	80	88	56	81	76	78	90	76

1.1 Represent the information above in a scatter plot

1.2 Determine the equation of the least squares regression line for this set of data

1.3 Hence, predict the final percentage for a learner obtaining 73% in the trial examination. Give your answer to the nearest percentage

1.4 Calculate the correlation coefficient for the above data

1.5 Do you think by using the least squares regression line one can accurately predict a learner's final percentage? Provide Mathematical justification for your answer.

QUESTION 2

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

2.1 Represent the information above in a scatter plot

2.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$

2.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100kJ

2.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 2.1

2.3 Identify an outlier in the data

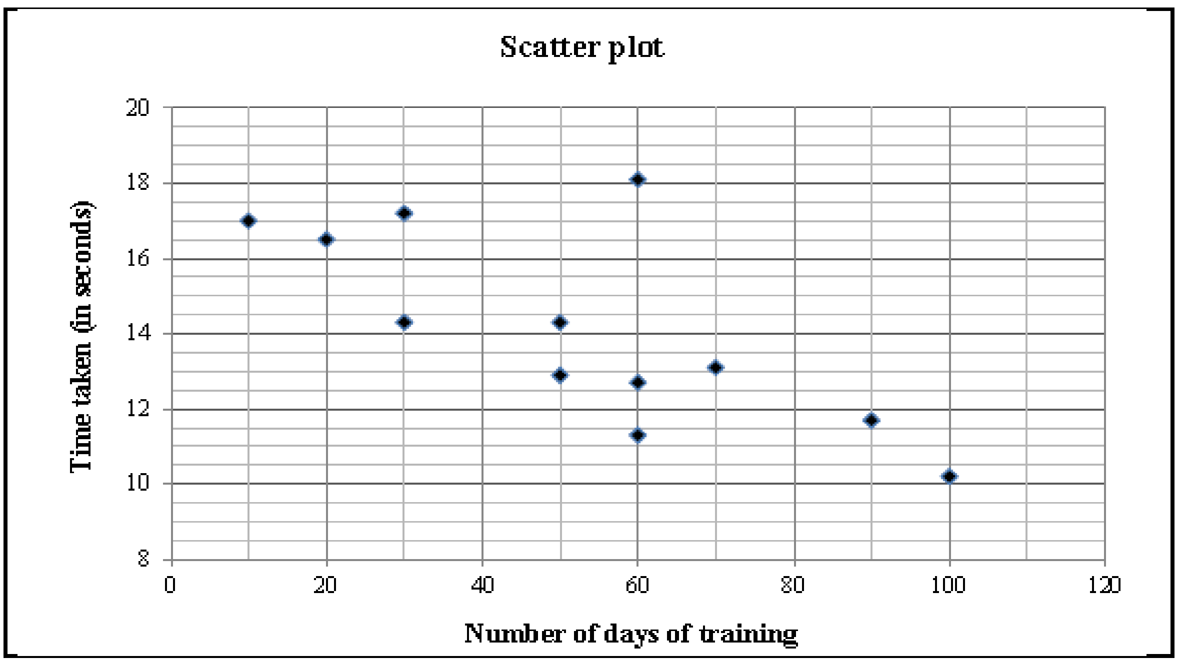
2.4 Calculate the value of the correlation coefficient

2.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy

QUESTION 3

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



3.1 Discuss the trend of the data collected

3.2 Identify any outlier(s) in the data

3.3 Calculate the equation of the least squares regression line

3.4 Predict the time taken to run the 600 m sprint for an athlete training for 45 days

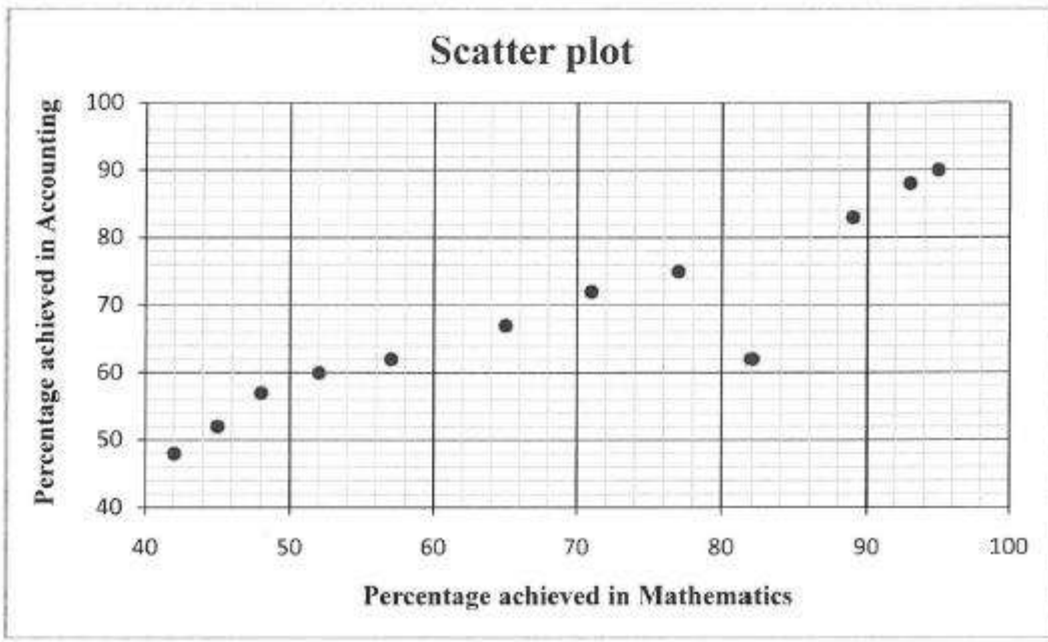
3.5 Calculate the correlation coefficient

3.6 Comment on the strength of the relationship between the variables

QUESTION 4

At a certain school, only 12 candidates take Mathematics and Accounting. The marks as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62



4.1 Calculate the mean percentage of the Mathematics data

4.2 Calculate the standard deviation of the Mathematics data

4.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean

4.4 Calculate an equation for the least squares regression line (line of best fit) for the data

4.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in QUESTION 4.4 (Round off your answer to the NEAREST INTEGER)

4.6 Use the scatter plot and identify any outlier(s) in the data

QUESTION 5

A group of 30 learners randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

5.1 Calculate the mean of the data

5.2 Determine the median of the data

5.3 Determine the standard deviation of the data

5.4 Determine the number of times that the sum of recorded values of the dice is within ONE standard deviation from the mean. Show your calculations

QUESTION6

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday

Time (hours)	Cumulative frequency
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

6.1 Draw an ogive (cumulative frequency curve) on the DIAGRAM SHEET to represent the above data

6.2 Write down the modal class of the data

6.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time

6.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday

QUESTION 7

The time taken (to the nearest minute) for a certain task to be completed was recorded on 48 occasions and the following data was obtained:

Time (hours)	**Cumulative frequency**
$11 \leq t < 15$	6
$15 \leq t < 19$	15
$19 \leq t < 23$	28
$23 \leq t < 27$	40
$27 \leq t < 31$	48

7.1 Write down the modal class

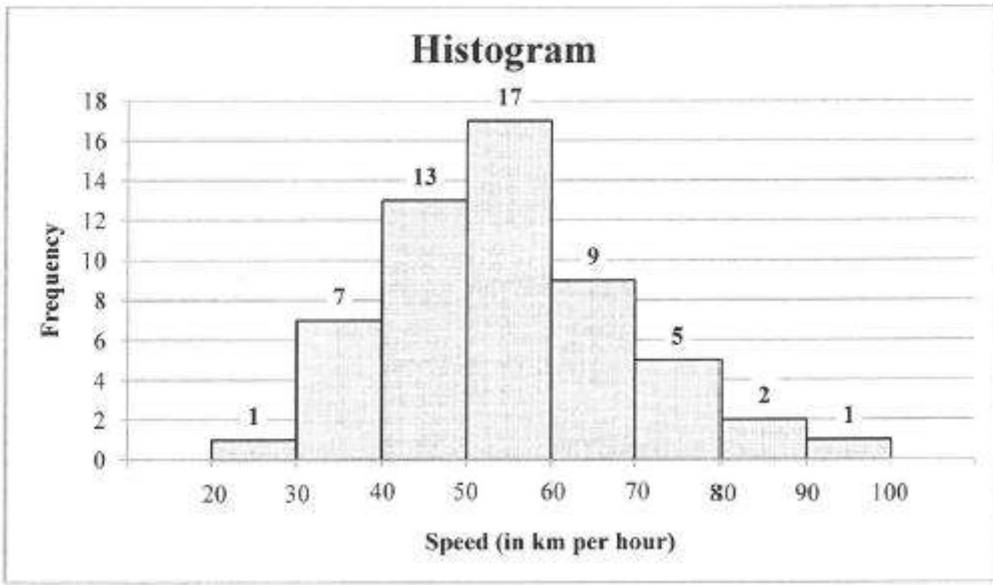
7.2 Draw an ogive (cumulative frequency curve) for the given data

7.3 Determine, using the ogive, the interquartile range for the data

7.4 Use your graph to estimate in how many of the 48 occasions was this task completed in more than 20 minutes?

QUESTION 8

The speed of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



8.1 Identify the modal class of the data

8.2 Use the histogram:

8.2.1 Complete the cumulative frequency column in the table on the DIAGRAM SHEET

8.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid on the DIAGRAM SHEET

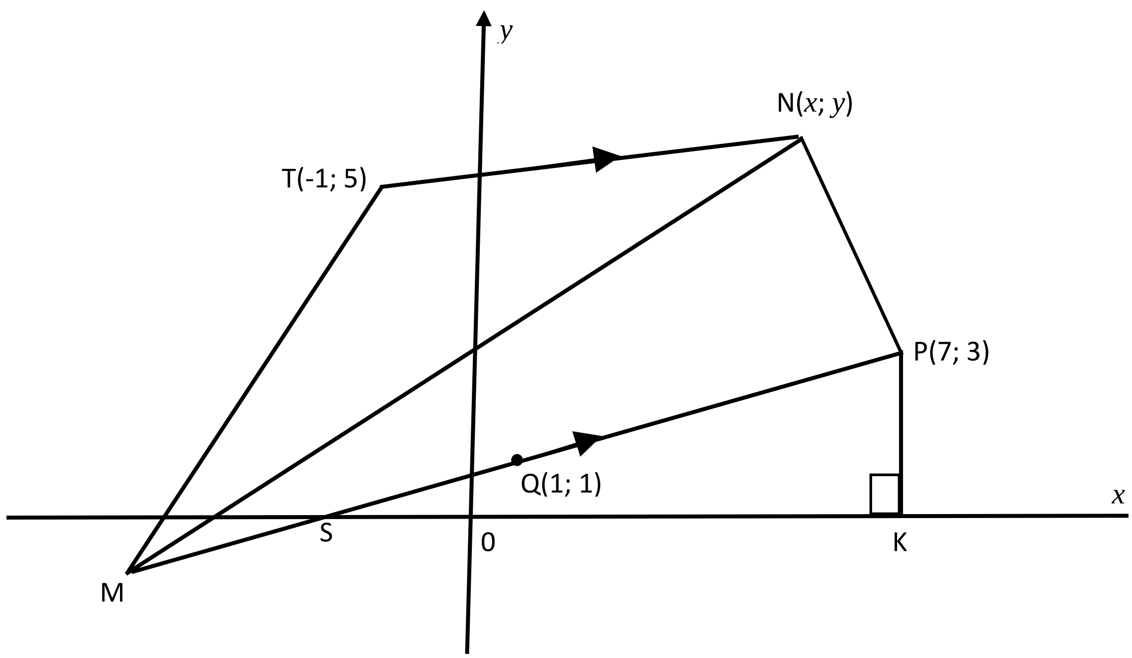
8.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorist who will receive a speeding fine.

ANALYTICAL GEOMETRY

- Almost all formulae to be used are in the formula sheet. (Refer)
- Knowledge of properties of triangles and quadrilaterals is key.
- Emphasize correct substitutions.
- Different formulas of the equation of a circle to be revised.
- Relationships between a tangent and a radius to be clarified.
- Advice learners on how to find the coordinates of a point.

QUESTION 1

In the diagram below, M, T (-1; 5), N(x; y) and P(7; 3) are vertices of trapezium MTNP having TN $\parallel$ MP. Q (1; 1) is the midpoint of MP. PK is a vertical line and $S\hat{P}K = \theta$. The equation of NP is $y = -2x + 17$



1.1 Write down the coordinates of K

1.2 Determine the coordinates of M

1.3 Determine the gradient of PM

1.4 Calculate the size of θ

1.5 Hence, or otherwise, determine the length of PS

1.6 Determine the coordinates of N

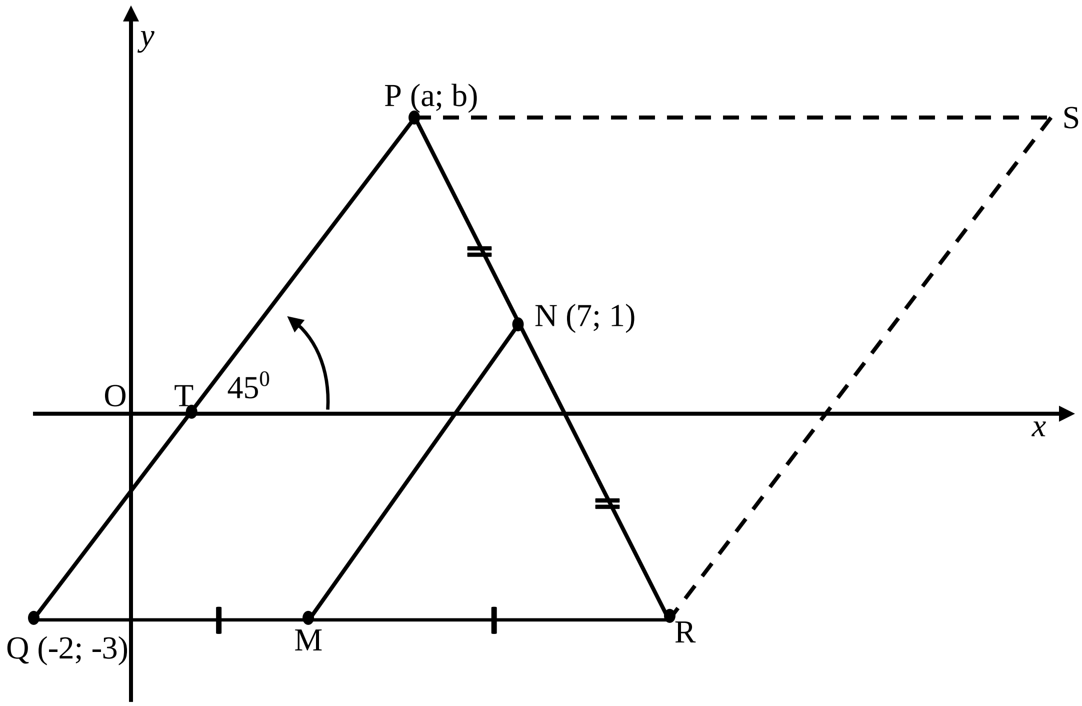
1.7 If A(a; 5) lies in the Cartesian plane:

    1.7.1 Write down the equation of the straight line representing the possible positions of A

    1.7.2 Hence, or otherwise, calculate the value(s) of a for which $T\hat{A}Q = 45^0$

QUESTION 2

In the diagram below, the line joining Q(-2; -3) and P(a; b), *a* and*b*> 0, makes an angle of 45^0 with the positive *x*-axis. QP = $7\sqrt{2}$ units. N(8; 1) is the midpoint of PR and M is the midpoint of QR.



Determine:

2.1 The gradient of PQ

2.2 The equation of MN in the form $y = mx + c$ and give reasons

2.3 The length of MN

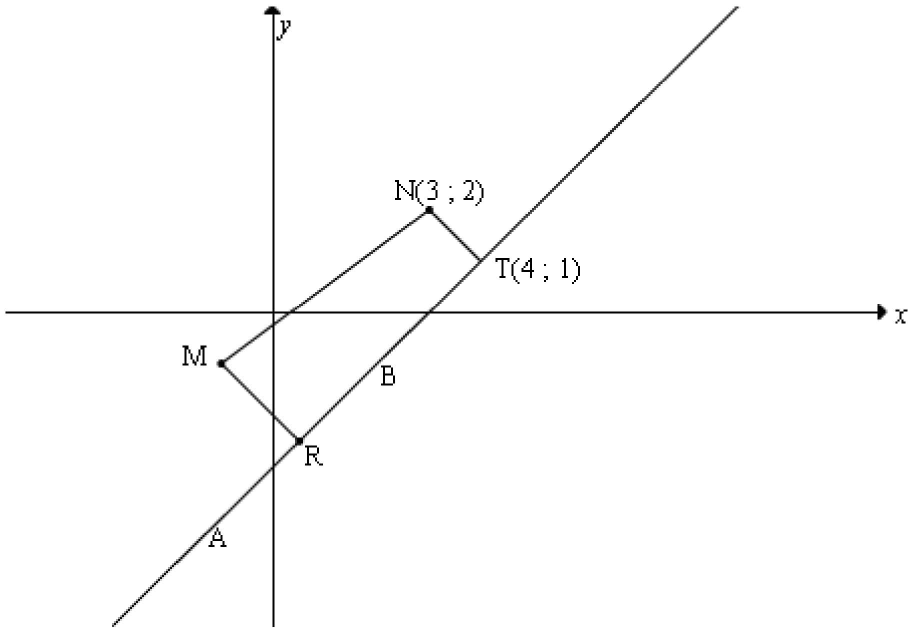
2.4 The length of RS

2.5 The coordinates of S such that PQRS, in this order, is a parallelogram

2.6 The coordinates of P

QUESTION 3

In the diagram below, the equation of the circle having centre M is $(x+1)^2+(y+1)^2=9$. R is a point on a chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N (3 ; 2) at point T (4; 1)



3.1 Write down the coordinates of M

3.2 Determine the equation of AT in the form $y=mx+c$

3.3 If it is further given that MR= $\frac{\sqrt{10}}{2}$ units, calculate the length of AB

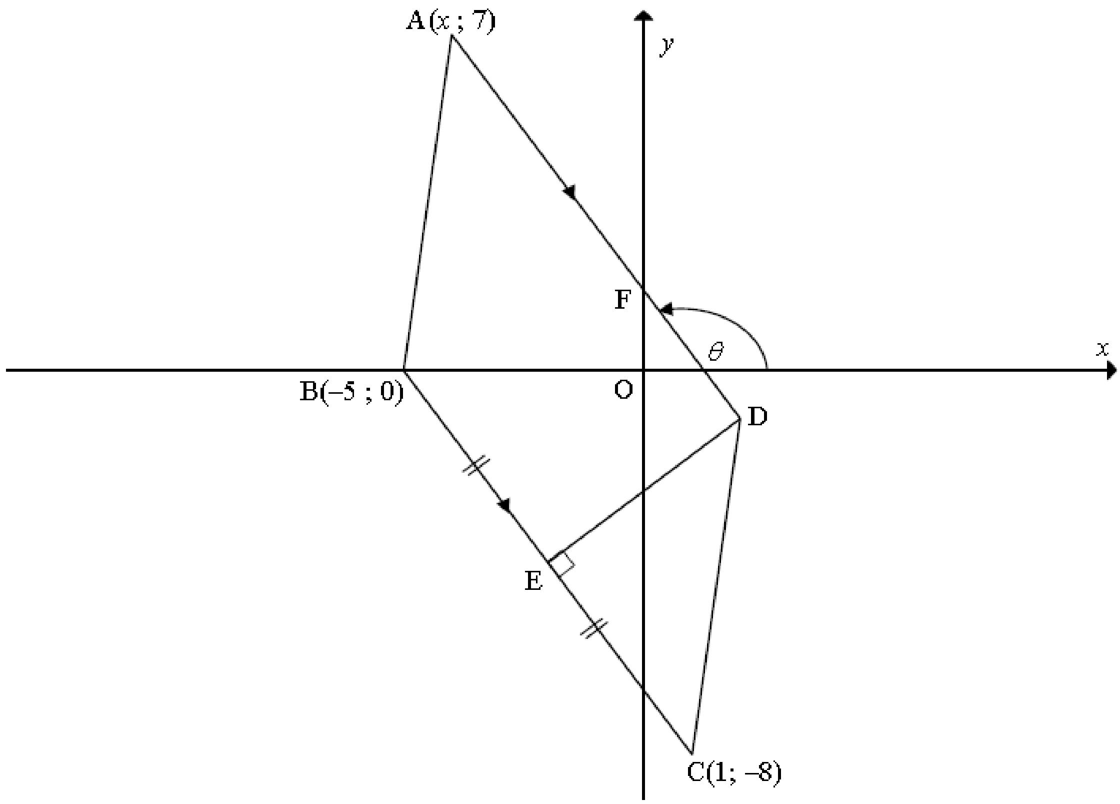
Leave your answer in simplest surd form

3.4 Calculate the length of MN

3.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2+y^2+Cx+Dy+E=0$

QUESTION 4

In the diagram, ABCD is a trapezium with AD $\parallel$ BC and vertices A(x; 7), B (-5; 0), C (1; -8) and D. DE $\perp$ BC with E on BC such that BE = EC. The inclination of AD with the positive x-axis is θ and AD cuts the y-axis in F.



4.1 Calculate the gradient of BC

4.2 Calculate the coordinates of E

4.3 Determine the equation of DE in the form $y = mx + c$

4.4 Calculate the size of θ

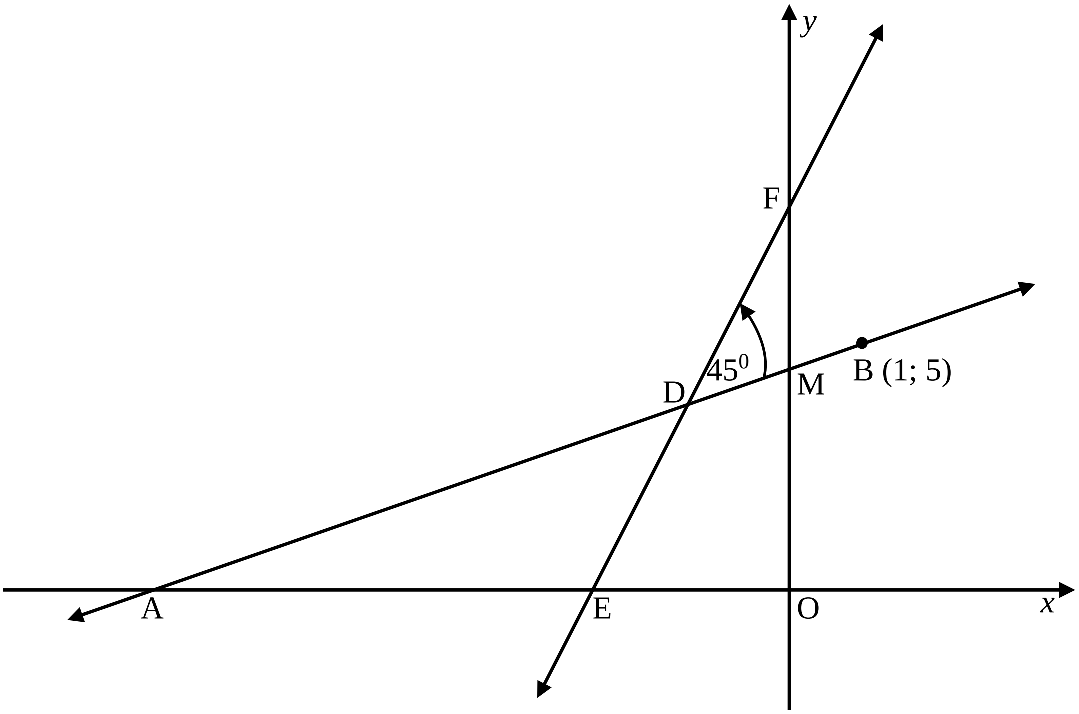
4.5 Calculate the size of $O\hat{F}D$

4.6 Calculate the value of x if the length of AB = $5\sqrt{2}$

4.7 Determine the equation of the circle with diameter BC in the form $(x-a)^2 + (y-b)^2 = r^2$

QUESTION 5

In the diagram below, E and F respectively are the *x*- and *y*-intercepts of the line having equation $y = 3x + 8$. The line through B(1; 5) making an angle of 45^0 with EF, as shown below, has *x*- and *y*- intercepts A and M respectively.



5.1 Determine the coordinates of E

5.2 Calculate the size of $D\hat{A}E$

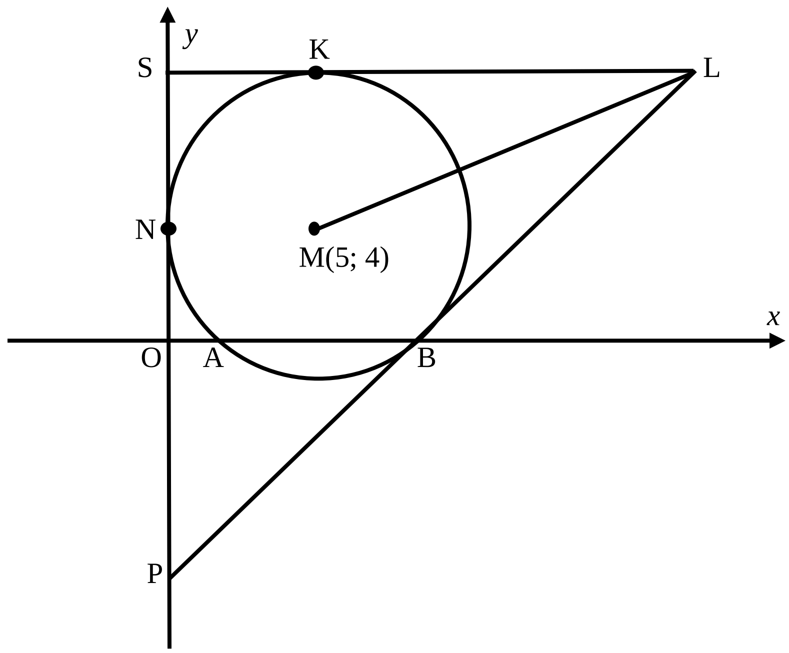
5.3 Determine the equation of AB in the form $y = mx + c$

5.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D

5.5 Calculate the area of quadrilateral DMOE

QUESTION 6

In the diagram below, a circle with centre M (5; 4) touches the y-axis at N and intersects the *x*-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the *x*-axis and P and S are points on the *y*-axis. LM is drawn.



6.1 Write down the length of the radius of the circle having centre M

6.2 Write down the equation of the circle having centre M, in the form

$(x-a)^2+(y-b)^2=r^2$

6.3 Calculate the coordinates of A

6.4 If the coordinates of B are (8; 0), calculate:

6.4.1 The gradient of MB

6.4.2 The equation of the tangent PB in the form $y=mx+c$

6.5 Write down the equation of tangent SKL

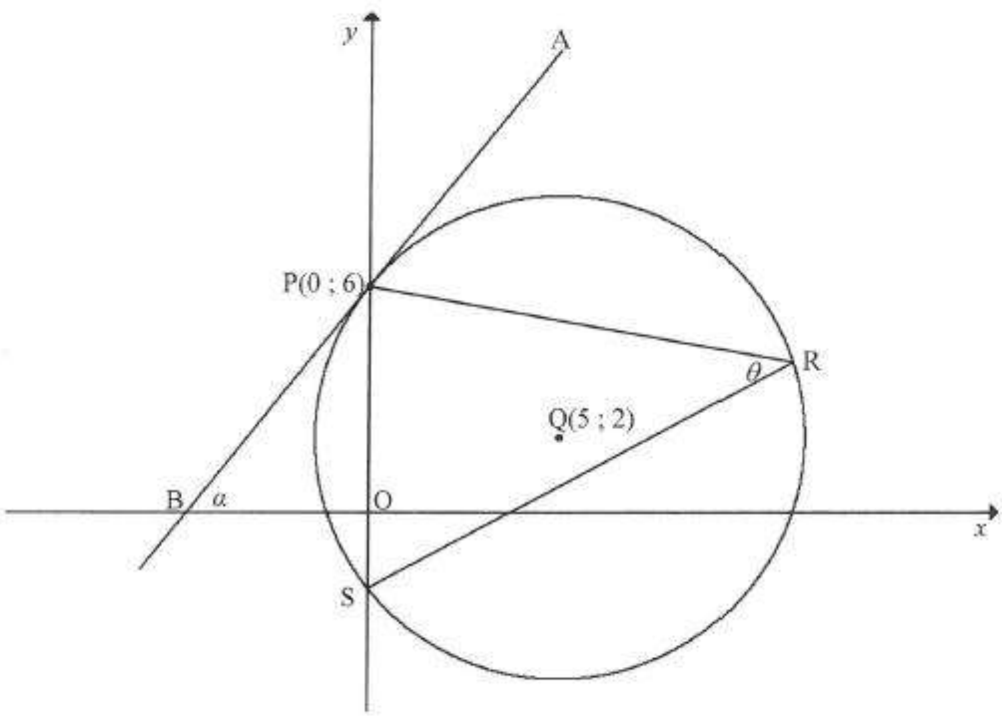
6.6 Show that L is the point (20; 9)

6.7 Calculate the length of ML in surd form

6.8 Determine the equation of the circle passing through point K, L and M in the form $(x-p)^2+(y-q)^2=c^2$

QUESTION 7

In the diagram below, Q (5; 2) is the centre of a circle that intersects the *y*-axis at P(0; 6) and S. The tangent APB at P intersects the *x*-axis at B and makes the angle α with the positive *x*-axis. R is a point on the circle and $P\hat{R}S = \theta$



7.1 Determine the equation of the circle in the form ($(x-a)^2 + (y-b)^2 = r^2$

7.2 Calculate the coordinates of S

7.3 Determine the equation of the tangent APB in the form $y = mx + c$

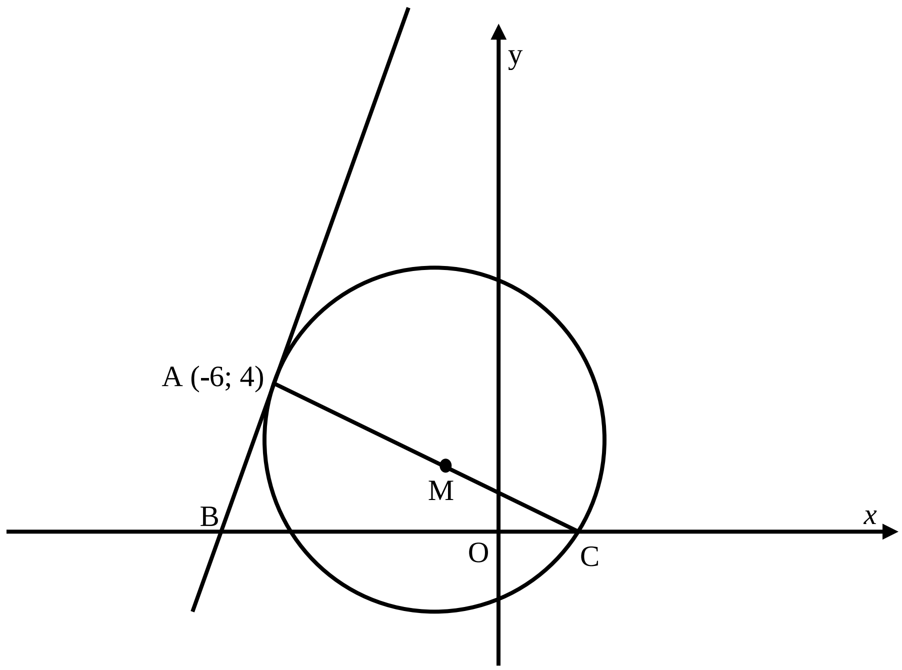
7.4 Calculate the size of α

7.5 Calculate, with reasons, the size of θ

7.6 Calculate the area of ΔPQS

QUESTION 8

In the diagram, the circle with centre M and equation $x^2 + y^2 + 4x - 4y - 12 = 0$ is drawn. C is the *x*-intercept of the circle. The tangent AB touches the circle at A (-6; 4) and cuts the *x*-axis at B.



8.1 Calculate the
  8.1.1 coordinates of M
  8.1.2 coordinates of C

8.2 Determine, giving reasons, the equation of the tangent AB in the form $y = mx + c$ if it is given that the gradient of MC = $-\frac{1}{2}$

8.3 Calculate the area of Δ ABC

8.4 Determine for which values of *k* the line $y = 2x + k$ will intersect the circle at two points

GENERAL TRIGONOMETRY

- Identifying the correct quadrant(PYTHAGORAS TYPE)
- Adherence to the instruction. Without using a calculator. Use a sketch diagram
- Correct selection and application of reduction formulae
- Identities
  - ✓ Prove type
  - ✓ Values where the identity is (in)valid
- Trig Equations
  - ✓ General solution
  - ✓ Solution in specific intervals
- CAST rule
- Understanding of negative angles and angles greater than 360^0
- Expansion of compound identities and the reverse process thereof

QUESTION 1

1.1 If $\cos 26 = r$, determine the following in terms of r in its simplest form:

1.1.1 $\cos 52^0$

1.1.2 $\tan 71^0$

1.2 Simplify to a single trigonometric ratio of A:

$$\frac{\tan(180^0 + A).\cos(180^0 - A).\sin(360^0 - A)}{\cos(90^0 - A)}$$

1.3 Prove the following identities:

1.3.1 $\dfrac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$

1.3.2 $2\cos^2(45^0 - A) = 1 + \sin 2A$

QUESTION 2

2.1 If $\sin 31^0 = p$, determine the following, without using a calculator, in terms of p:

2.1.1 $\sin 149^0$

2.1.2 $\cos(-59^0)$

2.1.3 $\cos(422^0)$

2.2 Simplify the following expression to a single trigonometric ratio:

2.2.1 $\tan(180^0 - \theta).\sin^2(90^0 + \theta) + \cos(\theta - 180^0).\sin\theta$

2.2.2 $\cos 325^0.\sin 745^0 - \cos(-205^0)\cos 55^0$

2.2.3 $\frac{3}{2}\cos 240^0 + \frac{1}{4}\tan^2(-60^0) - 2\sin 1395^0$

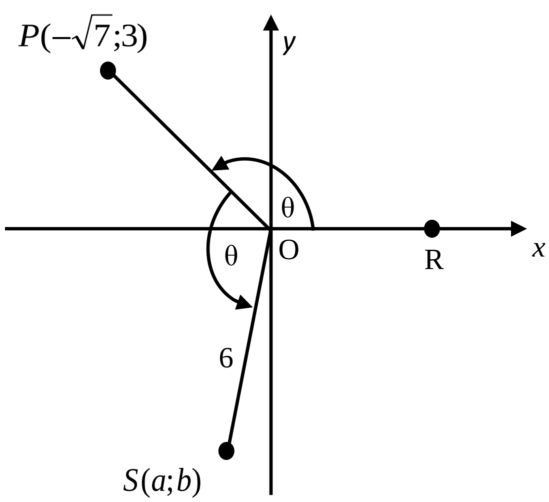
2.3 Consider: $\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

2.3.1 Prove the identity

2.3.2 Determine the values of x, where $x \in [180^0; 360^0]$, for which the above identity will be invalid/ undefined

QUESTION 3

$P(-\sqrt{7};3)$ and $S(a;b)$ are points on the Cartesian plane, as shown in the diagram below.
$P\hat{Q}R = P\hat{O}S = \theta$ and $OS = 6$



Determine, WITHOUT using a calculator, the value of:

3.1.1 $\tan\theta$

3.1.2 $\sin(-\theta)$

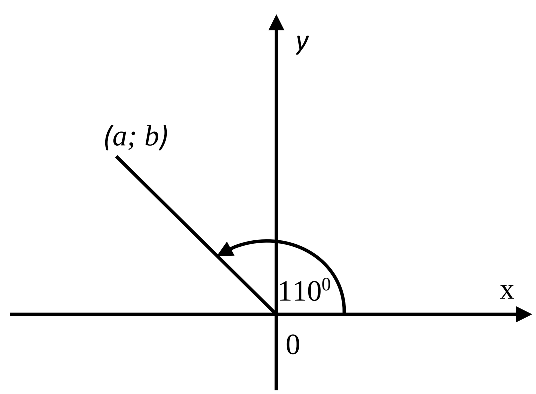
3.1.3 a

3.2 3.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio

3.2.2 Hence, calculate the value of $\dfrac{4\sin 15^0 \cos 15^0}{2\sin^2 15^0 -1}$ WITHOUT using a calculator. (Leave your answer in simplest form

QUESTION 4

4.1 In the Cartesian plane below, the point (a; b) and the angle of 110^0 are shown.



4.1.1 Complete the following: $\dfrac{b}{a} = ...110^0$

4.1.2 Determine, rounded off to two decimal places the value of :

$$\frac{b}{\sqrt{a^2+b^2}}$$

4.2 Given: $\sin 56^0 = q$

Determine without using a calculator, the value of the following in terms of q:

4.2.1 $\cos 146^0$

4.2.2 $\sin 112^0$

QUESTION 5

5.1 Given that $\cos\beta = -\frac{1}{\sqrt{5}}$, where $180^0 < \beta < 360^0$

Determine, with the aid of a sketch and without using a calculator, the value of $\sin\beta$

5.2 Determine the value of the following expression:

$$\frac{\tan(180^0 - x).\sin(x - 90^0)}{4\sin(360^0 + x)}$$

5.3 If sin A = p and cos A = q

5.3.1 Write tan A in terms of p and q

5.3.2 Simplify $p^4 - q^4$ to a single trigonometric ratio

5.4 Consider the identity: $\frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin\theta.\cos\theta} = \tan\theta$

5.4.1 Prove the identity

5.4.2 For which value(s) of θ in the interval $0^0 < \theta < 180^0$ will the identity be undefined?

5.5 Determine the general solution of $2\sin 2x + 3\sin x = 0$

QUESTION 6

6.1 Given that $\sin 23 = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator

6.1.1 $\sin 203^0$

6.1.2 $\cos 23^0$

6.1.3 $\tan(-23^0)$

6.2 Simplify the following expression to a single trigonometric function:

6.2.1 $$\frac{4\cos(-x).\cos(90^0 + x)}{\sin(30^0 - x).\cos x + \cos(30^0 - x).\sin x}$$

6.2.2 $$\frac{\sin 130^0 \tan 60^0}{\cos 540^0.\tan 230^0.\sin 400^0}$$

6.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$

6.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator.

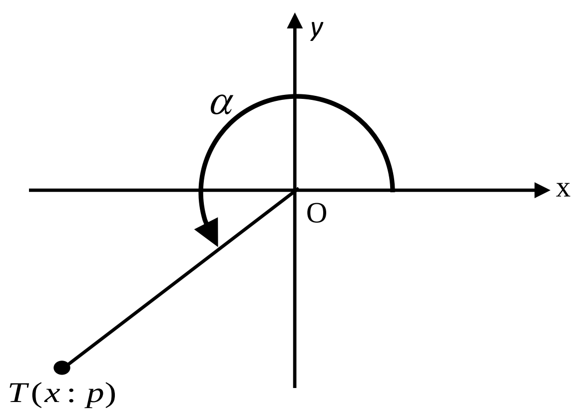
QUESTION 7

7.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, determine the value of $x^2 + y^2$

7.2 Simplify to a single term:

$\sin(540^0 - x).\sin(-x) - \cos(180^0 - x).\sin(90^0 + x)$

7.3 In the diagram below, $T(x : p)$ is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$



7.3.1 Show that $x = -1$

7.3.2 Write $\cos(180^0 + \alpha)$ in terms of p in its simplest form

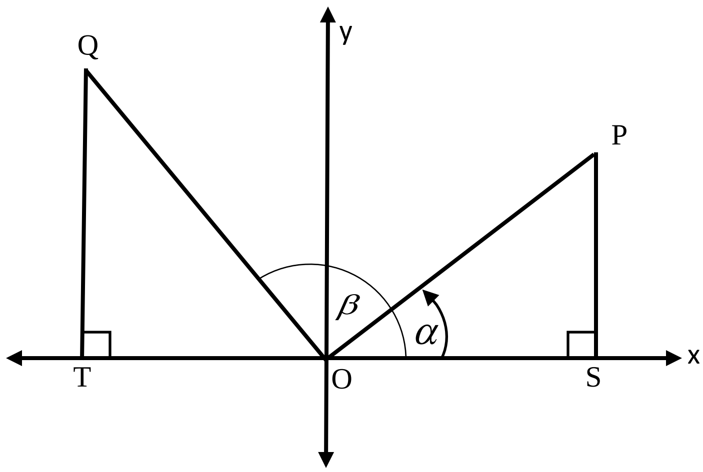
7.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$

7.4 7.4.1 For which value(s) of x will $\frac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0^0 \leq x \leq 180^0$?

7.4.2 Prove the identity: $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$

QUESTION 8

In the diagram below the equation of OP is given by $3y - 2x = 0$. S is a point on the x-axis is such that PS $\perp x$-axis. $S\hat{O}P = a$. The line segment OQ is drawn such that $S\hat{O}Q = \beta$. T is a point on the x-axis such that QT $\perp x$-axis.



8.1 Show that $\tan\alpha = \frac{2}{3}$

8.2 Calculate the value of $\sin\alpha$

8.3 Write down $Q\hat{O}P$ in terms of α and β

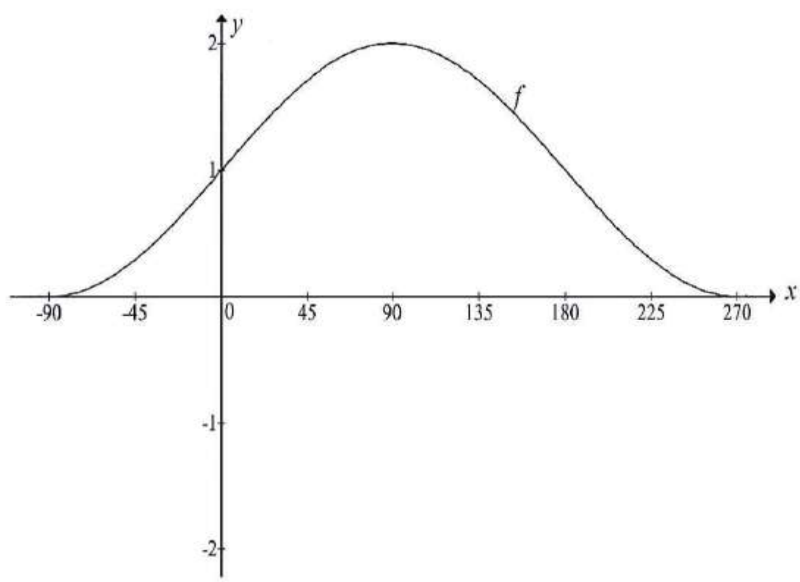
8.4 If it is given that $\sin\beta = \frac{3}{5}$, find the value of $Q\hat{O}P$

TRIGONOMETRY FUNCTIONS

- Drawing of sketch graphs
- Finding the defining equations
- Identifying the type of graph in a given sketch
- Characteristics of trig functions i.e. Amplitude, Period and Asymptote
- Transformations applied in trig graphs and their description in words.i.e. effects of parameters. (two at a time)
- Interval or restriction
- Correct use of notation. e.g. *f(x)>g(x)* versus *f(x)≥g(x)*

QUESTION 1

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90^0 \leq x \leq 270^0$



1.1 Write down the range of f

1.2 Show that $\sin x + 1 = \cos 2x$ can be written as $(2\sin x + 1)\sin x = 0$

1.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$

1.4 Use the grid on the DIAGRAM SHEET to draw the graph of $g(x) = \cos 2x$ for $-90^0 \leq x \leq 270^0$

1.5 Determine the value(s) of x for which $f(x+30^0)=g(x+30^0)$ in the interval $-90^0 \leq x \leq 270^0$

1.6 Consider the following geometric series:

$1+2\cos 2x+4\cos^2 2x+...$

Use the graph of g to determine the value(s) of x in the interval $0^0 \leq x \leq 90^0$ for which this series will converge.

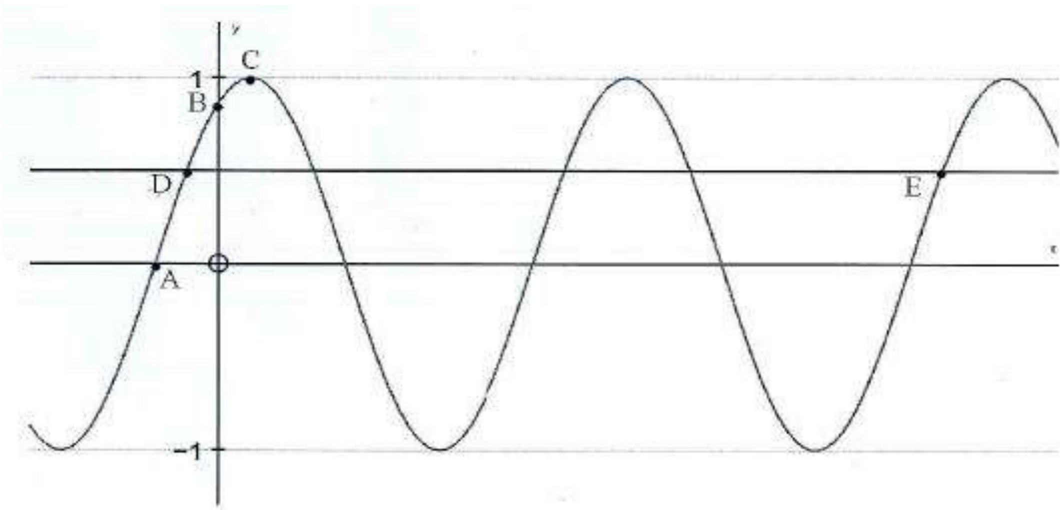
QUESTION 2

2.1 The graph $f(x)=2\cos(x-15^0)$ is given

2.1.1 Determine the amplitude of $f(x)$

2.1.2 If $h(x)= f(x)-2,$ determine the range of $h(x)$

2.2 Refer to the sketch graph of $y=\sin(2x+60^0)$ below. The line $y=0,5$ is sketched on the same set of axes. Points D and E represent the intersection of the two graphs and A and B are the x-and y -intercepts respectively. C is a maximum point on the sine graph.



Determine the co-ordinates of the following points, leaving your answers in surd form where necessary. Show all your working.

2.2.1 A

2.2.2 B

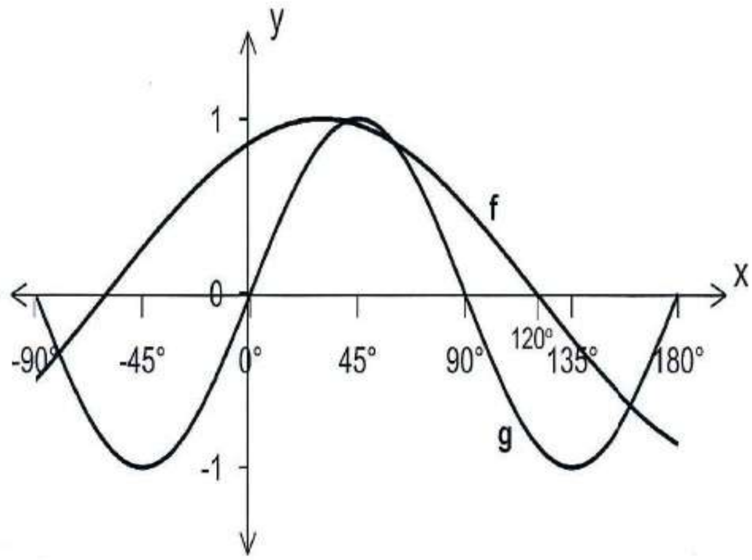
2.2.3 C

2.2.4 D

2.2.5 E

QUESTION 3

The graphs of $f(x) = \cos(x + a)$ and $g(x) = \sin bx$ are shown above for $x \in [-90^0; 180^0]$



3.1 Determine:

3.1.1 the value of *a*

3.1.2 the value of *b*

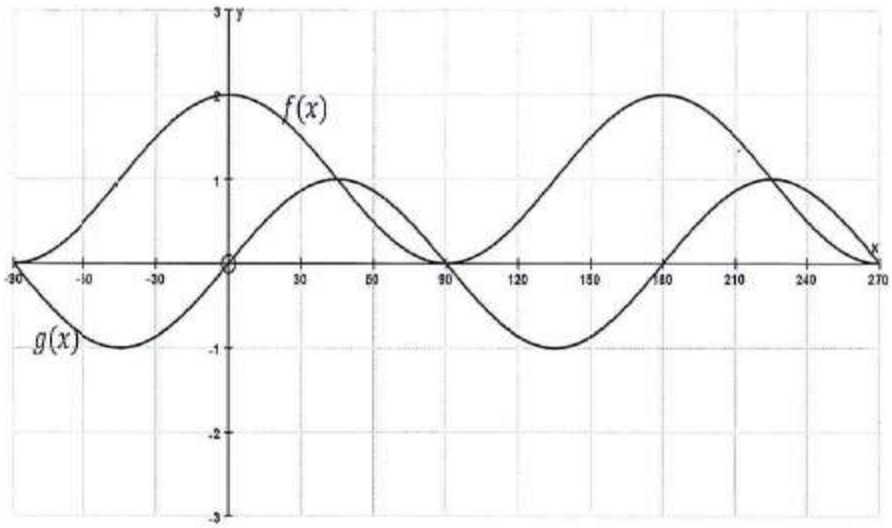
3.1.3 the amplitude of *f*

3.1.4 the period of *g*

3.2 If g is moved down 2 units, what will its equation change to?

QUESTION 4

In the diagram you are given that $f(x) = \cos bx + q$ and $g(x) = \sin cx$ for $x \in [-90^0; 270^0]$

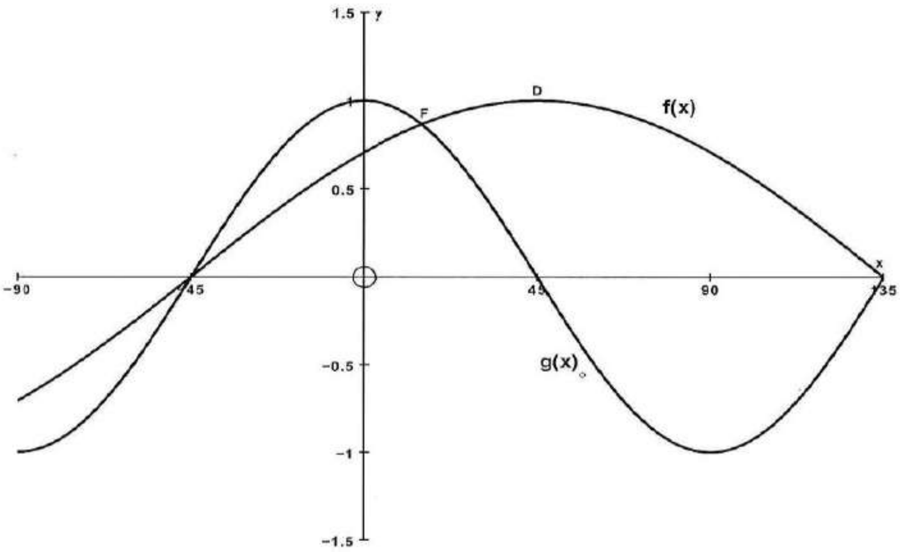


4.1 Determine the values of a, b, c and q

4.2 For which values of x is $f(x).g(x) > 0$?

QUESTION 5

This sketch shows the graphs of $f(x) = \sin(x + 45^0)$ and $g(x)$ both for $x\epsilon$ [90^0; 135^0]:

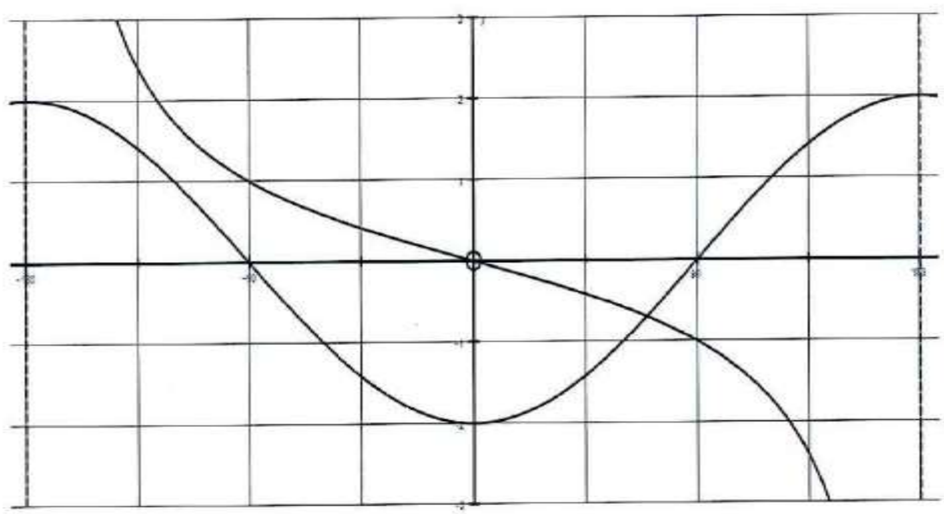


5.1 Write down the equation of $g(x)$

5.2 For which values of x is $g(x) > f(x)$ in the given interval, given that the x-coordinates of F is 15^0?

QUESTION 6

The graph below shows graphs of $f(x) = a\cos x$ and $h(x) = b\tan cx$ over the interval [-180; 180]. Use the graph to answer the questions that follow.



6.1 Find the values of a, b and c

6.2 Find a value for x $h(x) - f(x) = 2$

6.3 Find the values of x for which $f(x) \geq 0$

6.4 Find the value(s) of x for which $f(x)$ x $g(x) < 0$

QUESTION 7

7.1 7.1.1 If the period of $y = \tan 2x$ is halved, what is the new equation?

7.1.2 If the amplitude of $y = \cos 2x + 1$ is doubled, what is the new equation?

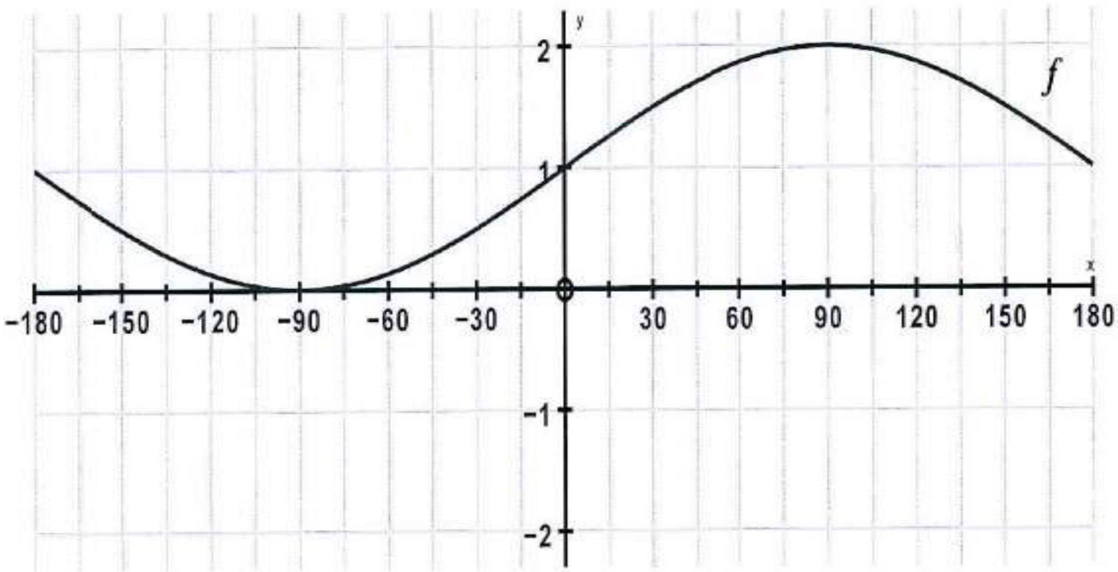
7.1.3 If the graph of $y = \sin(2x - 30^0)$ is translated left by 30^0, what is the new equation?

7.2 Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

7.2.1 Determine algebraically the values of x for which $f(x) = g(x)$ if $x \epsilon$ [-180^0; 180^0]

7.2.2 The graph of $f(x)$ is sketched below, for the interval $x \in [-180^0; 180^0]$

Sketch the graph of $g(x)$ on the same set of axes.



7.2.3 Use your graphs to write down the values of $x$for which $f(x).g(x) < 0$ in the givendomain

SOLUTION OF TRIANGLES

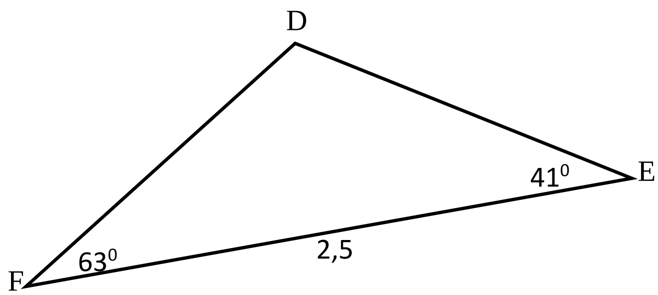
2Ds and 3Ds

Guide to examinations

- Identifying different planes in the sketch
- Integrating Euclid's Geometry
- Algebraic manipulation
- Minimum requirements for each triangle rule to be used

QUESTION 1

In the diagram alongside ΔDEF is drawn with $\hat{E} = 41^0$, $\hat{F} = 63^0$ and EF = 2, 5 units



Determine (rounded off to ONE decimal digit)

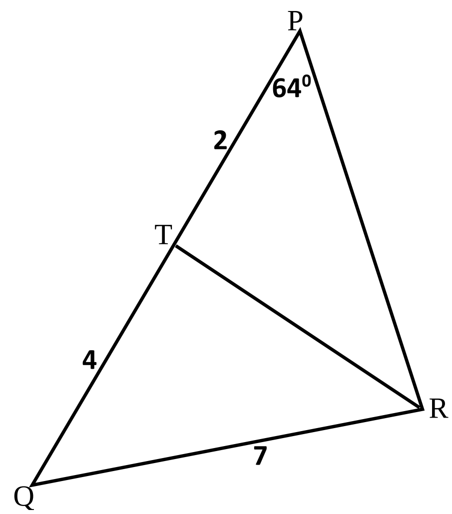
1.1 The length of DF

1.2 The area of ΔDEF

QUESTION 2

In the diagram alongside, ΔPQR is drawn with T on PQ

$\hat{P} = 64^0$, $QR = 7$ units, PT =2units and QT = 4 units



2.1 Calculate the size of $\hat{Q}$ (rounded off to the nearest degree)

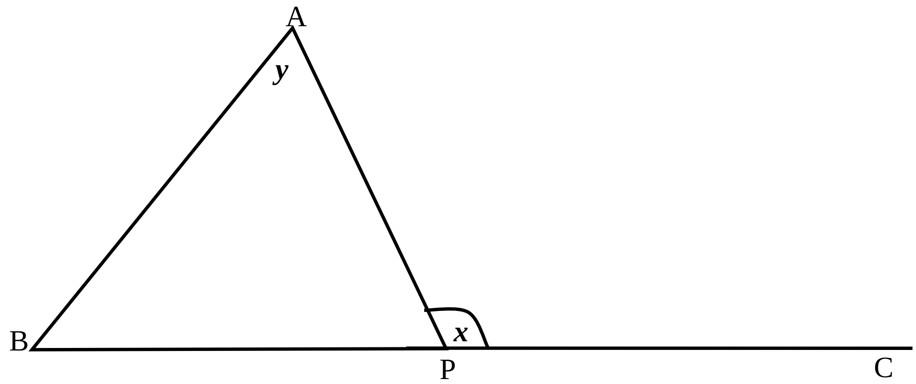
2.2 Hence, determine the following (rounded off to ONE decimal digit)

2.2.1 The area of ΔTQR

2.2.2 The length of ΔTR

QUESTION 3

In the diagram alongside, A, B, P and C are the positions of four players on a sports field



B, P and C are in a straight line, $A\hat{P}C = x$ and $\hat{A} = y$

3.1 Express $A\hat{P}B$ in terms of x

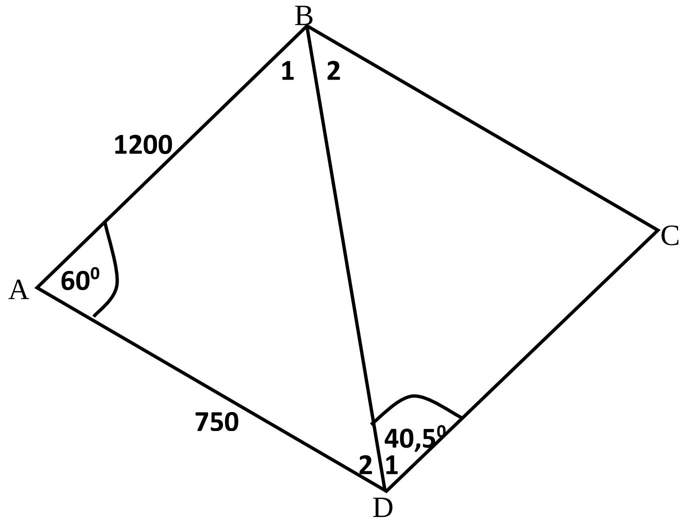
3.2 Prove that the distance between players A and B is given by $AB = \dfrac{BP.\sin x}{\sin y}$

3.3 If BP =50m, x = 150^0 and $\hat{B}$ =30^0, calculate, **without using a calculator** the distance AB. (Leave your answer in surd form)

QUESTION 4

Farmer Molefe has a piece of land in the form of a cyclic quadrilateral ABCD. The following measurements are given:

AB = 1 200m, AD = 750m, $\hat{A} = 60^0$ and $\hat{D}_1 = 40,5^0$



Determine:

4.1 The length of BD, rounded off to the nearest metre

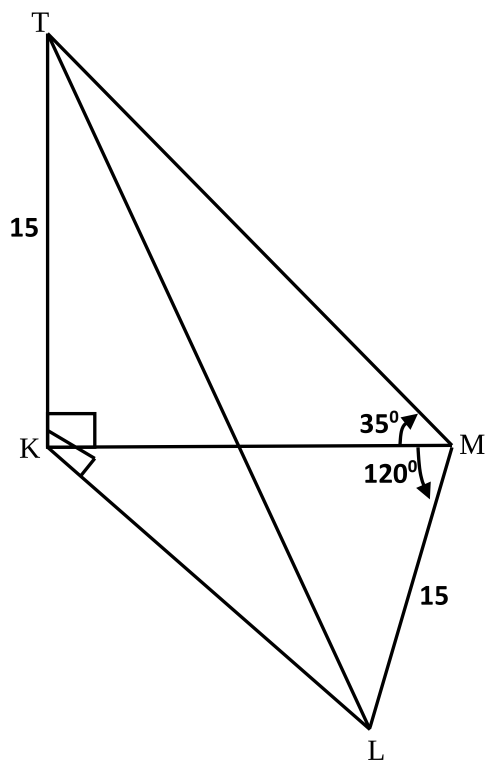
4.2 The size of $\hat{C}$

4.3 The length of BC

4.4 The area of ΔABD, rounded off to the nearest square metre

QUESTION 5

In the sketch below, K, L and M are three points in the same horizontal plane such that $K\hat{M}L = 120^0$. T represents a point vertically above K such that TK=LM=15cm and $T\hat{K}L = 90^0$.



5.1 Determine the length of KM

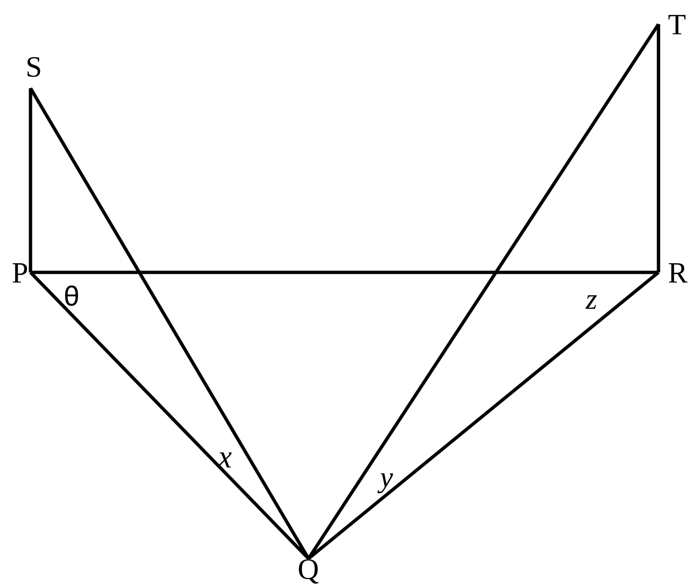
5.2 Show that the length of KL= 31; 7m (Show all your calculations)

5.3 Determine the size of $K\hat{T}L$

PROVE TYPE QUESTIONS

QUESTION 1

P, Q and R are three points on the same horizontal plane. PS and RT are the two vertical poles. Wires are strung from Q to the tops of the poles. The wire from Q to S forms an angle of x^0 with the ground. The other wire forms an angle of y^0 with the horizontal plane and is t metres long.



1.1 Show that $QR = t.\cos y$

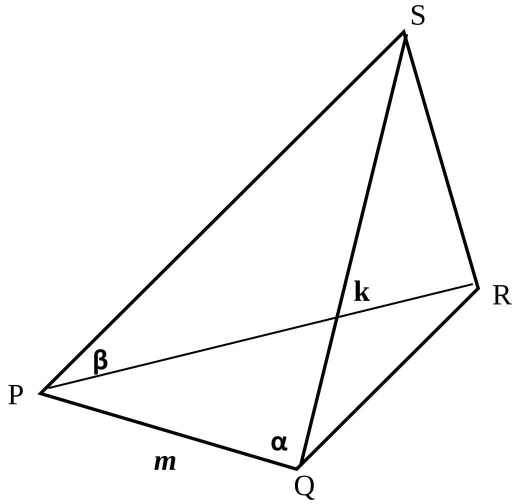
1.2 Show that $PQ = \dfrac{t.\cos y.\sin z}{\sin\theta}$

1.3 Prove that $PS = \dfrac{t.\sin z.\tan x.\cos y}{\sin\theta}$

QUESTION 2

In the figure SR is a vertical mast. P, Q and R are 3 points in the same horizontal plane. PS and QS are stay ropes. $PQ = m$; $QS = k$; $P\hat{Q}S = \alpha$. The angle of elevation of S from P is β.

If $k = 2m$, show that: $PS = m\sqrt{5 - 4\cos\alpha}$

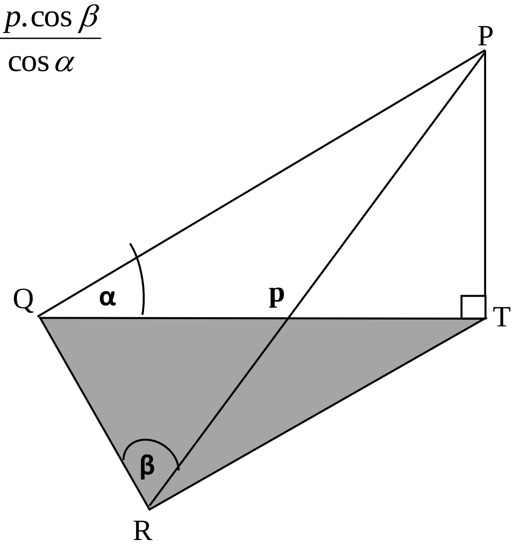


QUESTION 3

In the figure, Q, T and R are points in the horizontal plane and TP represents a vertical pole positioned at T. The angle of elevation of P from Q is α .

QT = p and PQ = PR. $P\hat{R}Q = \beta$

Prove that $PQ = \dfrac{2p.\cos\beta}{\cos\alpha}$



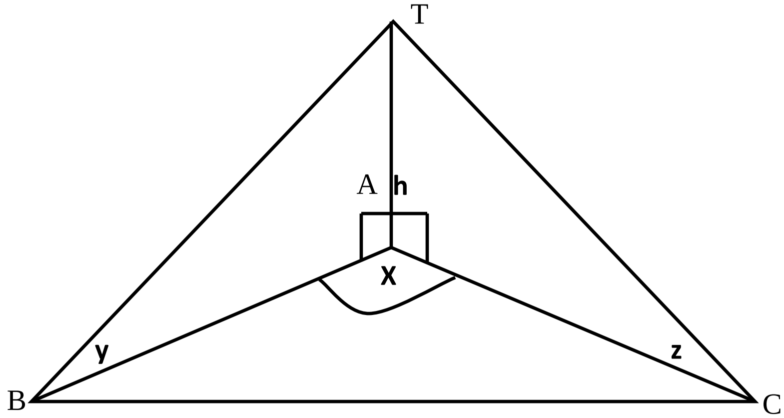
QUESTION 4

In the diagram below TA represents the vertical pole of height h erected in the horizontal plane ABC.

$A\hat{B}T = y$

$B\hat{A}C = x$

$A\hat{C}T = z$



4.1 Prove that:

Area $\Delta ABC = \dfrac{h^2 \sin x}{2\tan y.\tan z}$

4.2 Calculate the value of h if the area of $\Delta ABC = 51{,}8m^2$, $x = 123{,}7^0$, $y = 37{,}2^0$ and $z = 61{,}6^0$

EUCLID'S GEOMETRY

1. Explain the theorems and show the relationship in a diagram. Clearly indicate where the theorems does NOT hold.
2. Cover the basic work thoroughly
3. Use exploratory methods
4. Use the correct terminology i.e. acceptable reasons and the correct notation.
5. Focus on the following
   - Theory
   - Recognizing theorems in simple diagrams
   - Deconstructing a complex diagram to identify theorems
6. Name angles properly
7. For Grade 12 discuss the following theorems:
   - Midpoint theorem
   - Properties of quadrilaterals
   - Areas of triangles having equal heights and between same parallel lines
   - Explain both conditions for similarity
8. Spend more time teaching Euclid's Geometry in ALL grades

SOLVING RIDERS

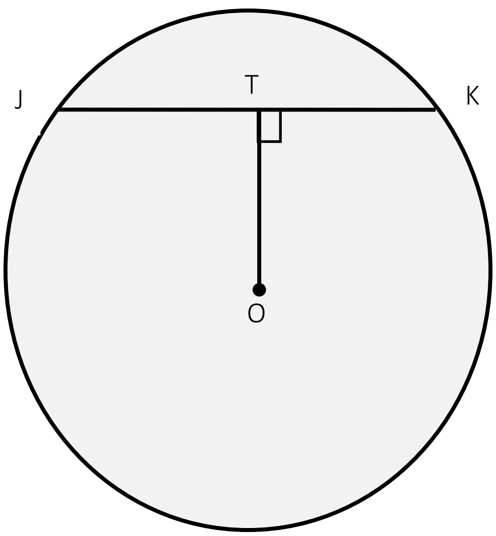
1. Scrutinize the given information
2. Identify the condition that must be used to solve the problem
3. Follow statement, reason approach. Statements must be accompanied by reasons
4. Refrain from making assumptions
5. Do not write correct but irrelevant statements
6. Check the link between questions
7. Use the acceptable abbreviations for reasons as prescribed

QUESTION 1

In the diagram below, JK is a chord of the circle with centre O.

OT ⊥JK; JK = 6x units; OJ = 10 units and OT = x units

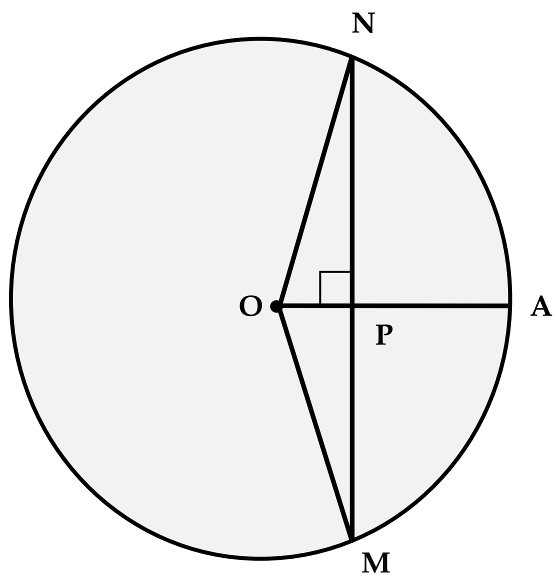
Determine, with reasons, the numerical value of x.



QUESTION 2

In the diagram below, O is the centre of circle NAM and

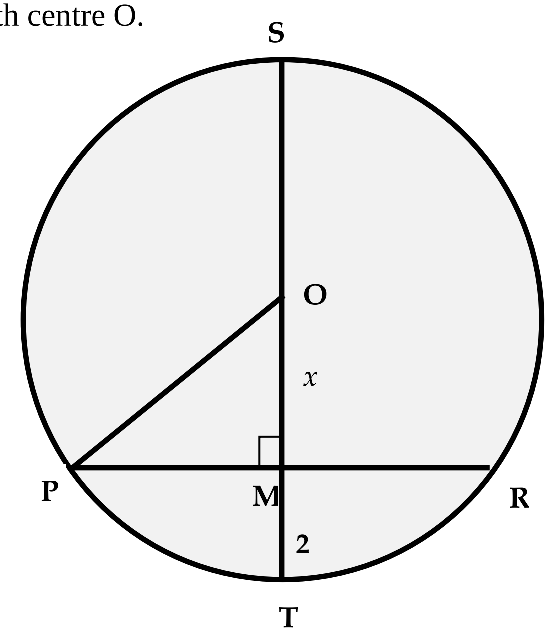
OPA ⊥ MPN.
MN = 48 units
OP = 7 units



Calculate, with reasons, the length of PA.

QUESTION 3

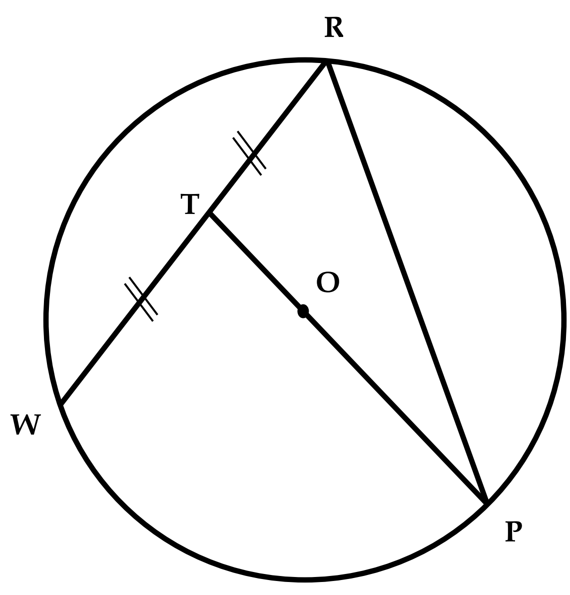
In the diagram below, PR is a chord of the circle with centre O.
Diameter ST is perpendicular to PR at M.
PR = 8 cm
MT = 2cm
OM = x cm



3.1 Write OP in terms of x and a number.
3.2 What is the length of PM?
3.3 Calculate the length of the radius of the circle.

QUESTION 4

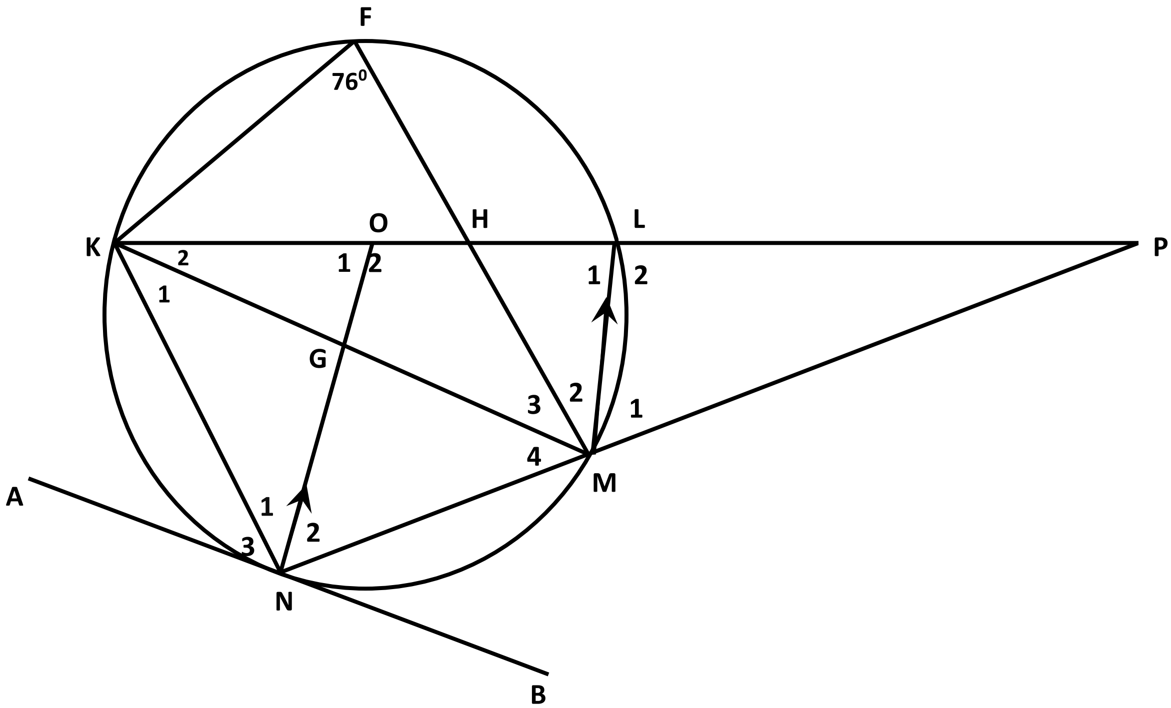
In the diagram below, O is the centre of circle WRP. Radius PO is produced to bisect chord WR at T. PR = 60 mm and PT = 40 mm



Calculate, giving reasons, the length of chord WR. (Leave the answer in simplified surd form.)

QUESTION 5

O is the centre of the circle and diameter KL is produced to meet at NM produced at P. ON ∥ LM. ANB is a tangent to the circle at N and $\hat{F} = 76^0$.



Use the DIAGRAM above to identify angles that are related to the following angles. Give reasons for the relationship. Then find the size of these angles.

5.1 $\hat{L}_1$
5.2 $\hat{O}_1$
5.3 $\hat{M}_4$
5.4 $\hat{N}_3$
5.5 $\hat{N}_1 + \hat{N}_2$
5.6 $\hat{N}_1 + \hat{N}_3$
5.7 $\hat{M}_1$
5.8 $\hat{M}_2 + \hat{M}_3$

QUESTION 6

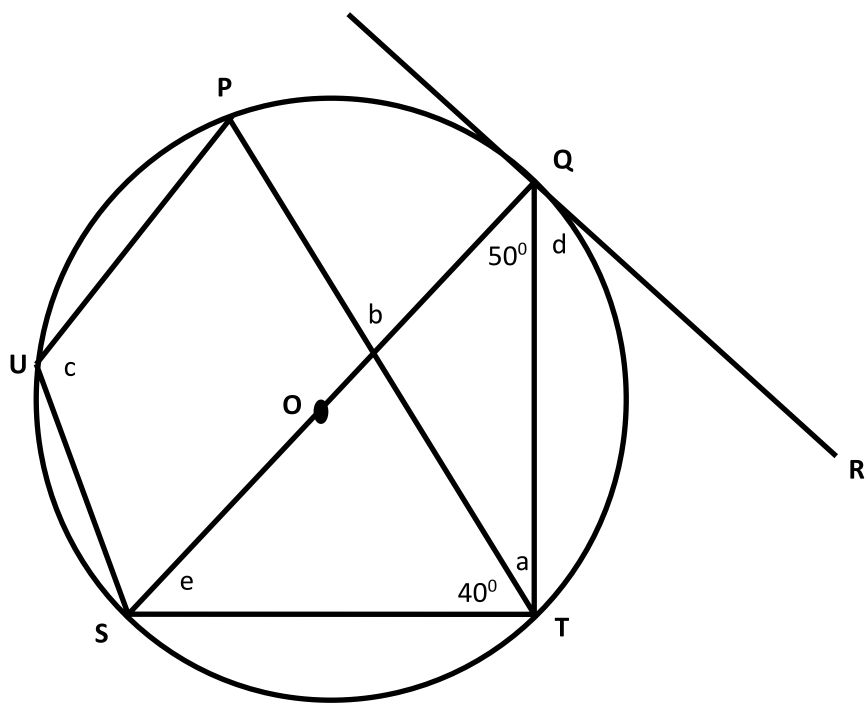
Refer to the diagram.

RQ is a tangent to circle $QTSUP$ with centre O. SOQ and PT are straight lines.

$P\hat{T}S = 40°$ and $SQT = 50°$

Find, with reasons, the size of the lower case letter marked (a) to (e).

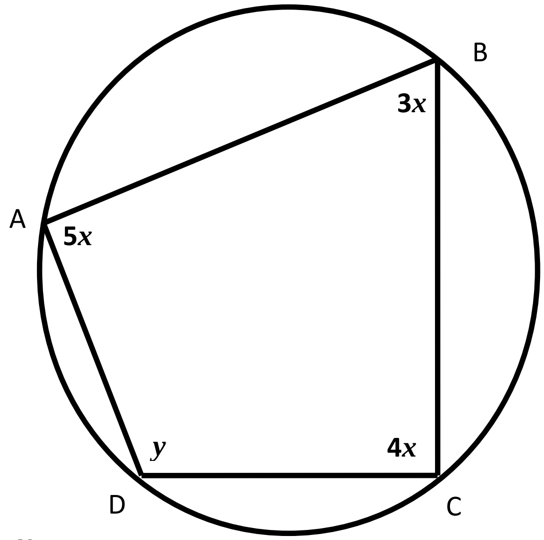
Fill in your answers in the table below.



Angle	Answer	Reason
a		
b		
c		
d		
e		

QUESTION 7

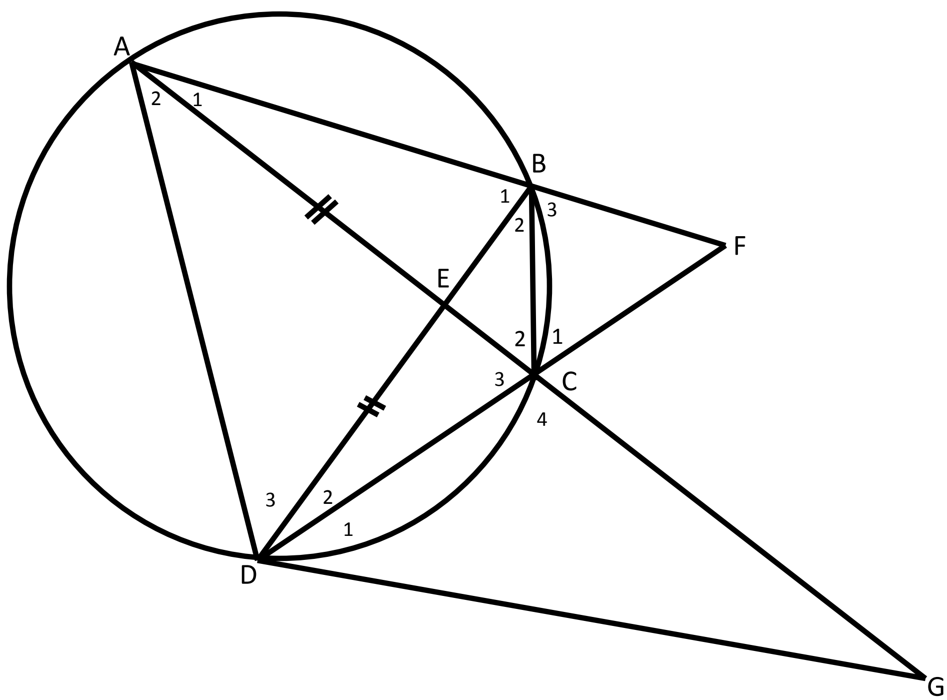
ABCD is a cyclic quadrilateral



Calculate the values of x and y

QUESTION 8

In the sketch below $ABCD$ is a cyclic quadrilateral the diagonals AC and BD intersect at E so that $AE = DE$. AB and DC both produced, meet at F .The tangent DG meet AC produced at G



$C\hat{D}G = 40°$ and $E\hat{D}C = 30°$

8.1 $\hat{A}_1$

8.2 $\hat{A}_2$

8.3 $\hat{C}_2$

8.4 $\hat{G}$

QUESTION 9

LOM is a diameter of circle *LMT*.

O is the centre of the circle.

$NL \perp NP$

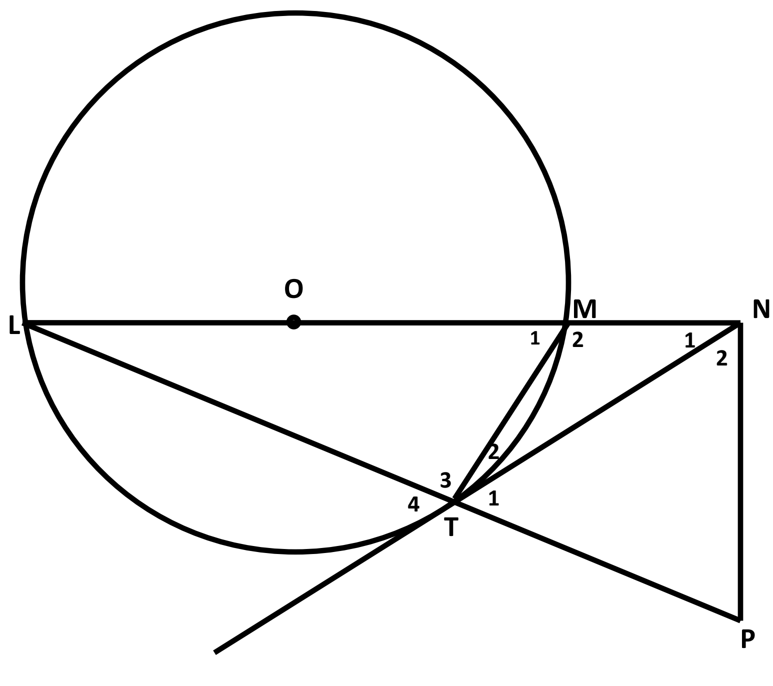
MT is a chord, NT is a tangent at T

LT is a chord produced to P.

Prove that:

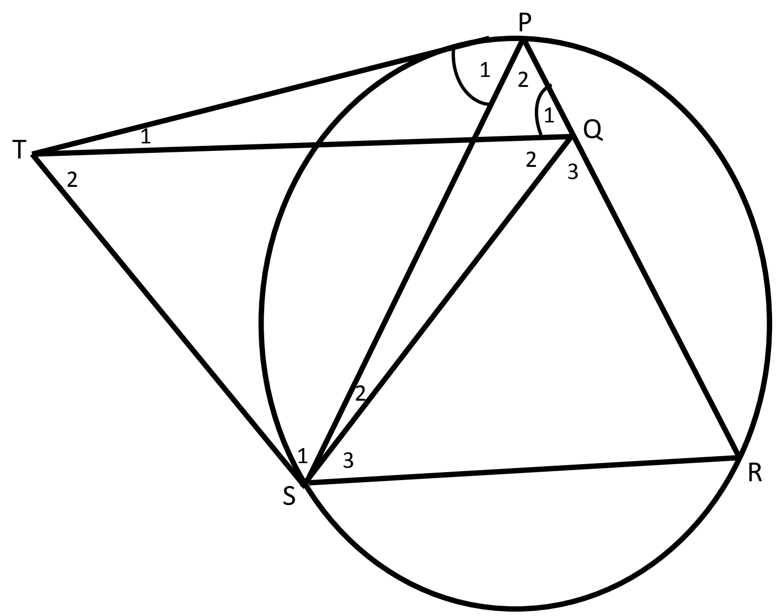
9.1 *MNPT* is a cyclic quadrilateral

9.2 $NP = NT$



QUESTION 10

In the figure below TP and TS are tangents to the circle. R is a point on the circumference. Q is a point on PR such that $\hat{Q}_1 = \hat{P}_1$, let $\hat{P}_1 = x$.



10.1 Show that $TQ // SR$

10.2 Prove that $QPTS$ is a cyclic quadrilateral.

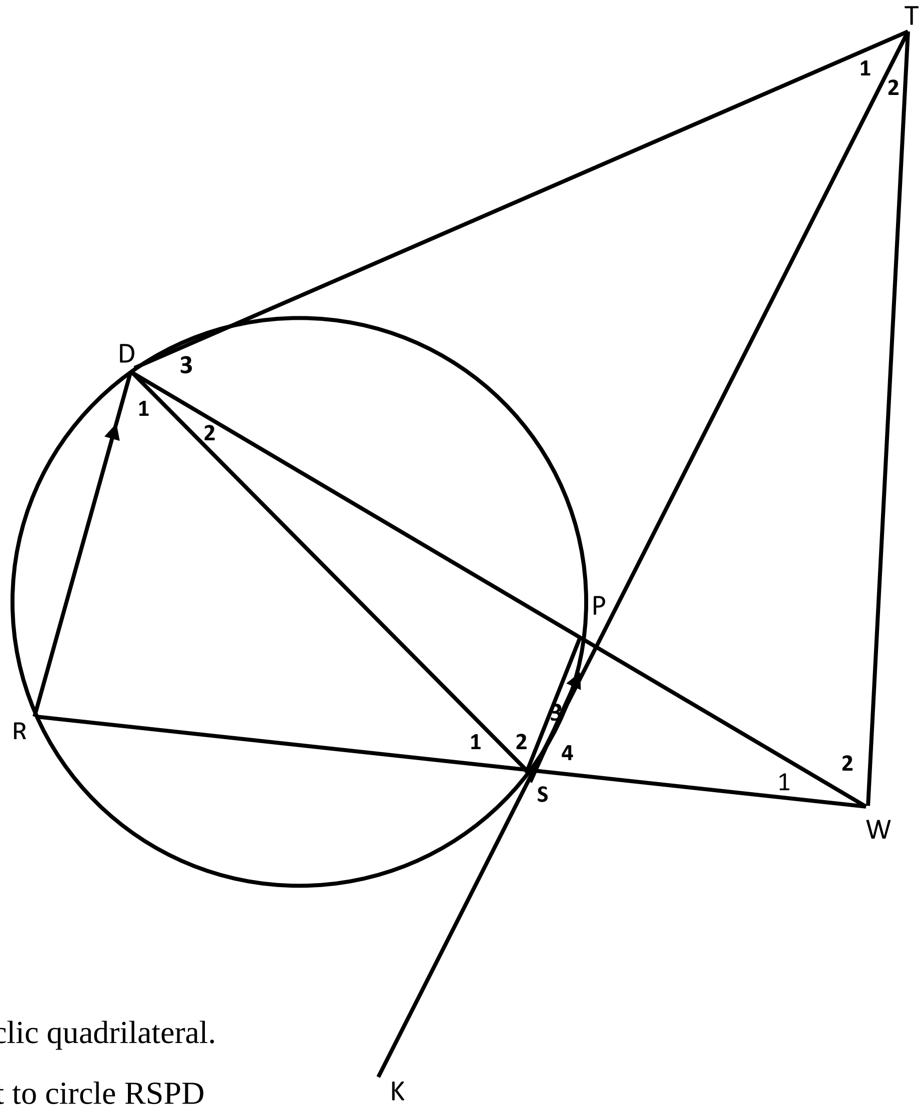
10.3 Prove that TQ bisects $S\hat{Q}P$

QUESTION 11

In the program, TD is a tangent to circle RSPD.RS and DP produced meet at W. KST is a straight line.

$\hat{S}_4 = \hat{S}_2$

$DR // PS$



Prove that:

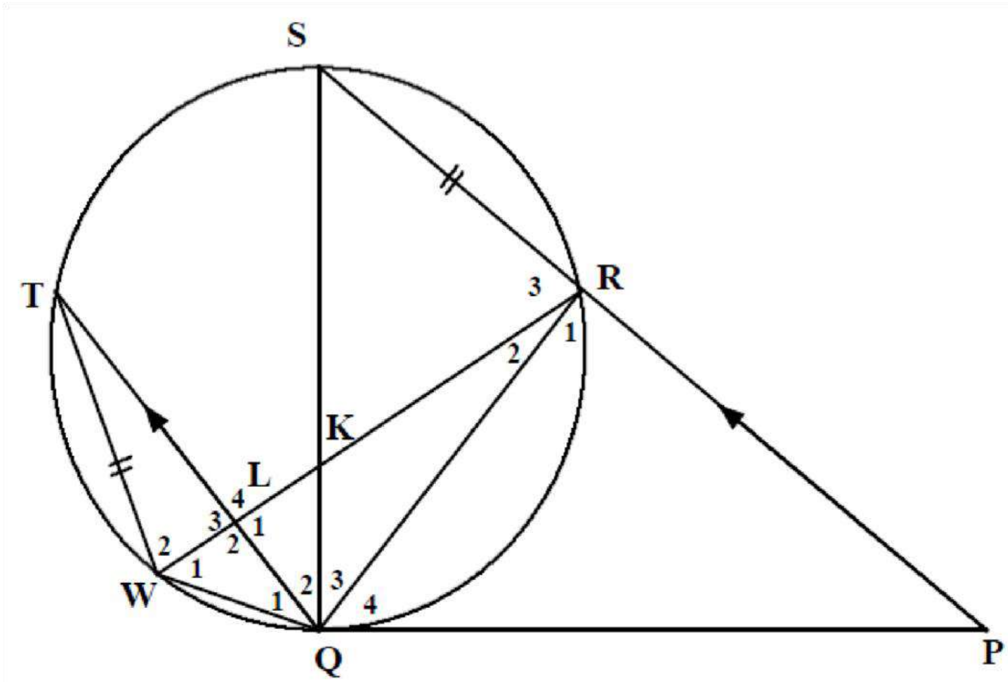
11.1 SWTD is a cyclic quadrilateral.

11.2 TSK is tangent to circle RSPD

11.3 $TW // PS$

QUESTION 12

In the diagram, PQ is a tangent to circle SRQWT at Q. PRS is a straight line. RW cuts at K and cuts Q at L. PS // QT;RS = TW



Prove that:

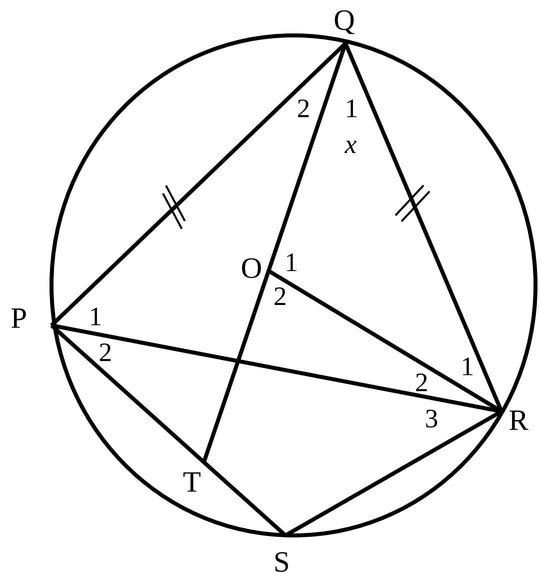
12.1 KQ is a tangent to circle LQW

12.2 $\hat{R}_1 = \hat{L}_3$

12.3 PRKQ is cyclic quadrilateral

QUESTION 13

In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS. PQ = QR and $\hat{Q}_1 = x$.



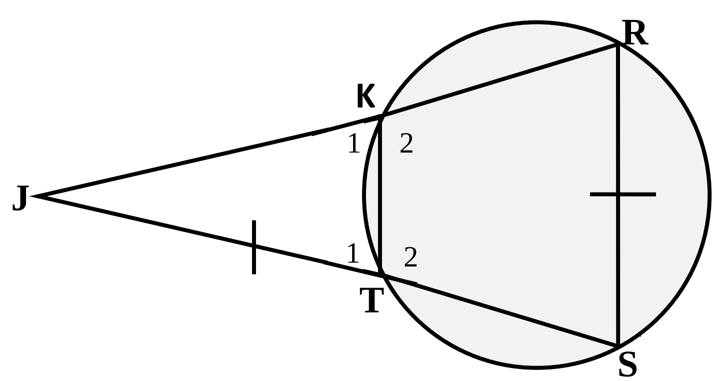
13.1 Calculate, with reasons, $\hat{P}_1$ in terms of x

13.2 Show that TQ bisects $P\hat{Q}R$

13.3 Show that STOR is a cyclic quadrilateral

QUESTION 14

In the diagram below, circle KRST is drawn. Chords RK and ST produced meet at J.JT = RS



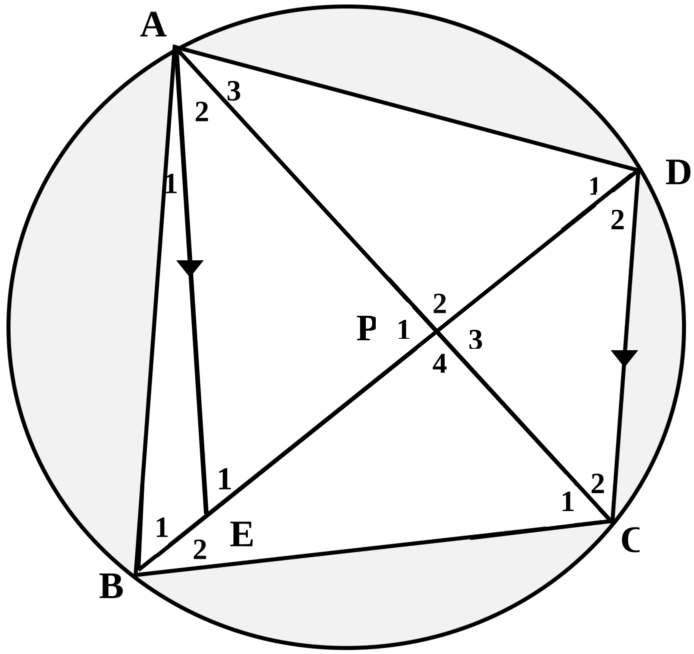
Prove that:

14.1 $\Delta JRS \,///\, \Delta JTK$

14.2 $\mathrm{JR}\,.\,\mathrm{TK} = RS^2$

QUESTION 15

In the diagram below, ABCD is a cyclic quadrilateral. AC and BD intersect at P. E is a point on BD so that AE //DC.



Prove that:

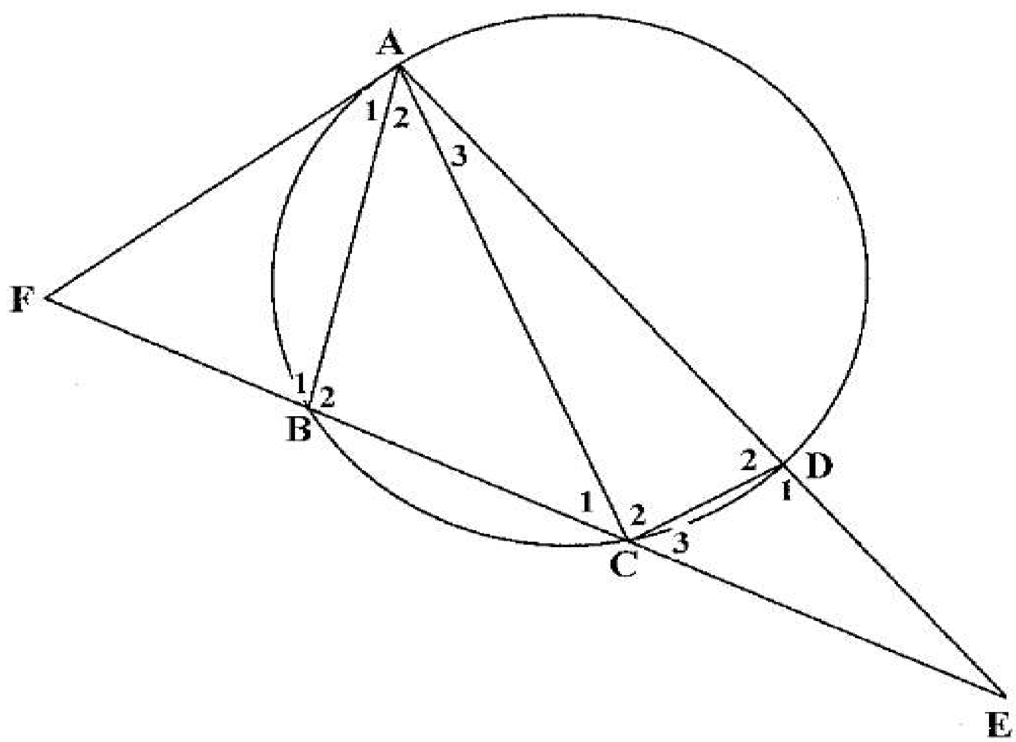
15.1 $\Delta APE /// \Delta BPA$

15.2 $AP^2 = BP \ . \ PE$

15.3 $\frac{AP}{PC} = \frac{PE}{PD}$

QUESTION 16

In the diagram below, FA is a tangent to circle ABCD at A. AD and FBC produced meet at E.



16.1 Prove that $\Delta DEC \,/\!/\!/\, \Delta BEA$

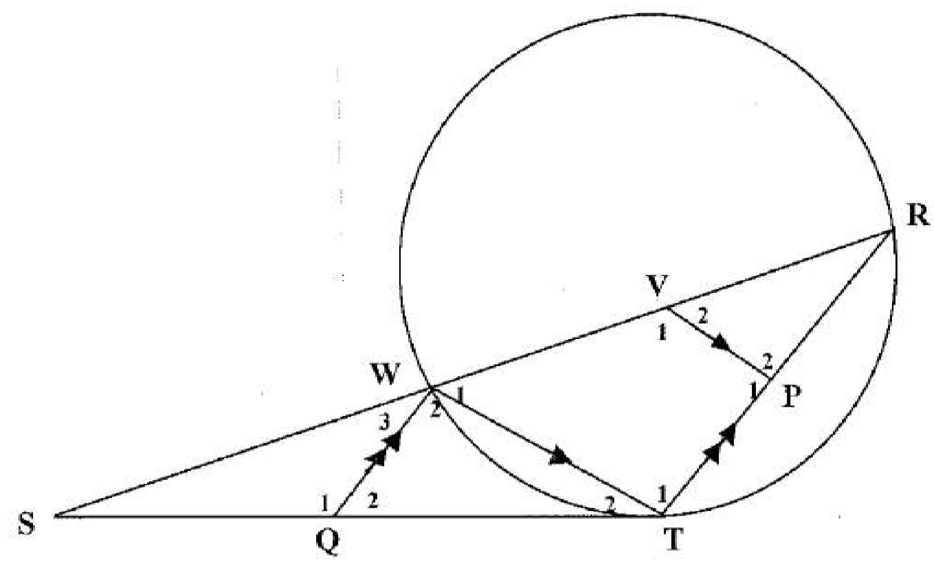
16.2 Prove that $\Delta FAB \,/\!/\!/\, \Delta FCA$

16.3 Hence, show that FA.CA = FC . AB

QUESTION 17

In the diagram below, ST is a tangent to circle RWT at T.

SWVR is a straight line. VP // WT with P on RT. WQ // RT with Q on ST.



17.1 Prove that ΔSTW /// ΔSRT.

17.2 Hence, write ST^2 in terms of the sides of ΔSTW and ΔSRT.

17.3 Hence, calculate the length of WR if ST = 6 cm and SW = 4 cm.

17.4 Name, without stating reasons, **ONE** other pair of similar triangles in the diagram.

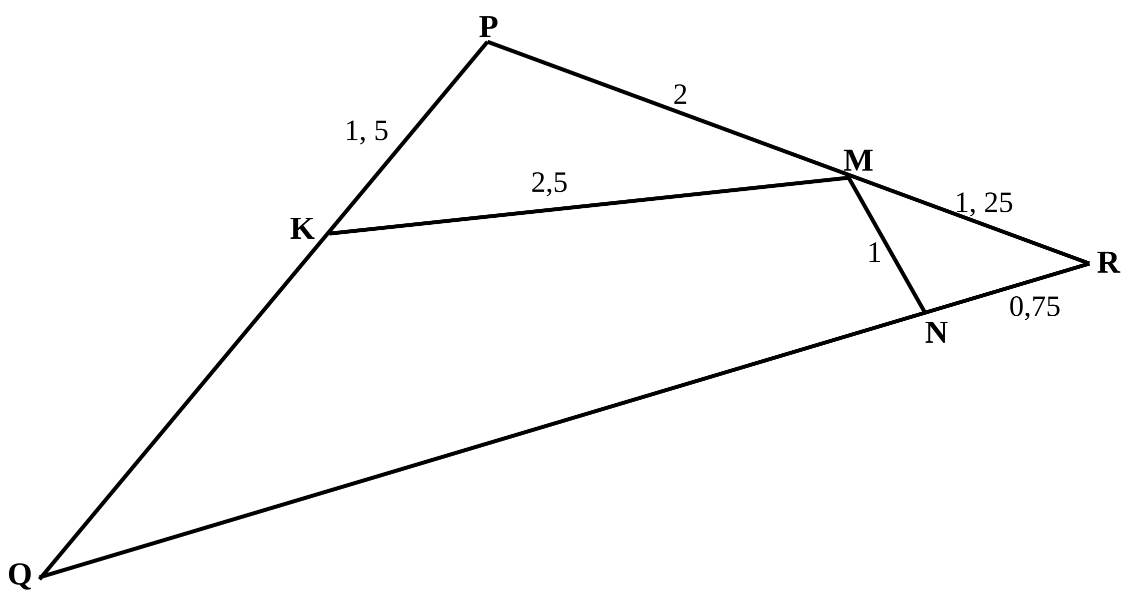
17.5 Hence or otherwise, determine the numerical value of $\frac{RP}{PT}$ if VR = 2 cm.
State reason(s).

QUESTION 18

18.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are …..

18.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1.25 and NR = 0, 75.

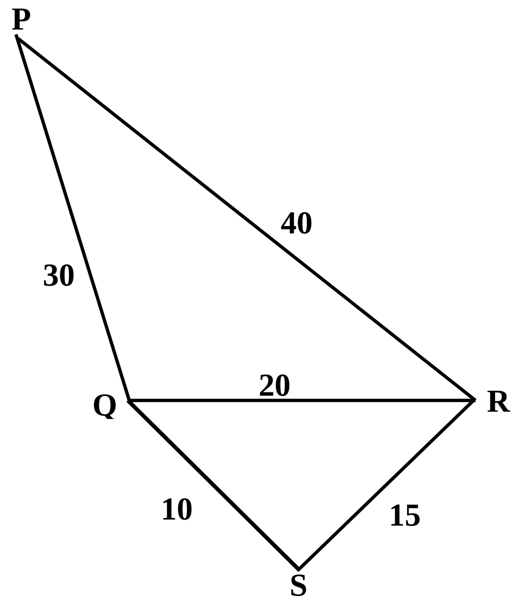


18.2.1 Prove that ΔKPM ||| ΔRNM.

18.2.2 Determine the length of NQ.

QUESTION 19

In the diagram below, PQ = 30 units, QS = 10 units, SR = 15 units, RP = 40 units and QR = 20 units.



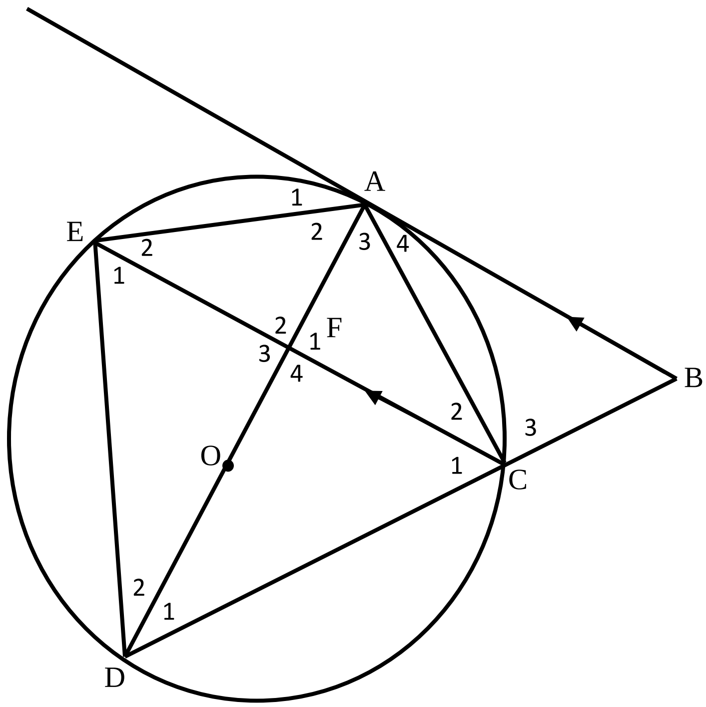
Prove that:

19.1 $\Delta PQR /// \Delta RSQ$

19.2 QS // PR

QUESTION 20

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA ∥ CE. DC produced, cuts tangent BA at B.



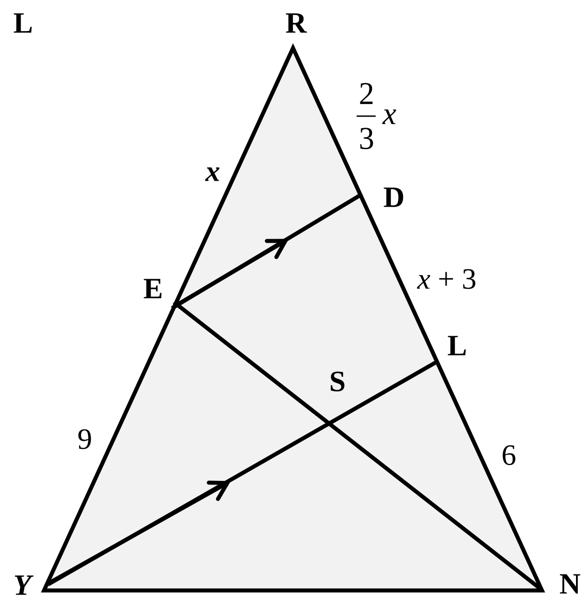
20.1 Show $\hat{C}_2 = \hat{D}_1$

20.2 Prove that ΔACF ||| ΔADC

20.3 Prove that AD = 4AF

QUESTION 21

NL = 6 units. RE = x units, RD = $\frac{2}{3}x$ units. EY = 9 units and DL = $x + 3$ units. S is a point on YL and ED // YL



21.1 Calculate the value of x

21.2 Hence or otherwise, show that L is the midpoint of DN

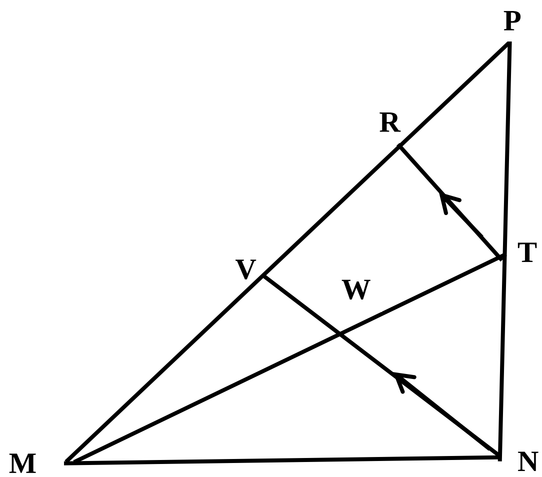
21.3 If SL = 1,4 units **write** down the length of DE

21.4 If Area of ΔRED = 2,7 units determine the Area of ΔREN.

QUESTION 22

In the figure below ΔPMN has V as the midpoint of PM. RT // VN

$\frac{TN}{PT} = \frac{4}{7}$



Determine the following:

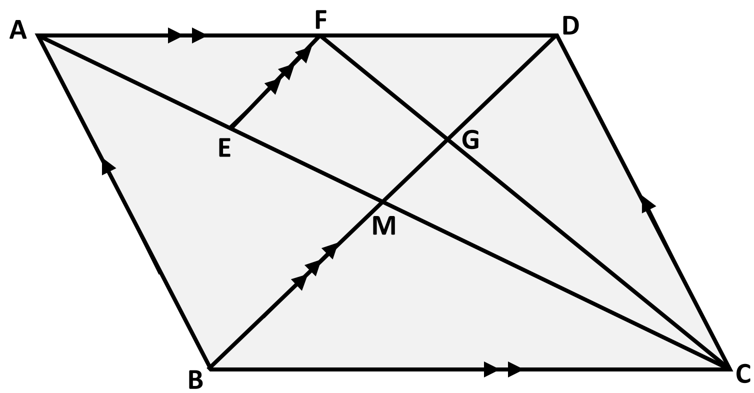
22.1 $\frac{PR}{PV}$

22.2 $\frac{PM}{RV}$

22.3 $\frac{MW}{WT}$

QUESTION 23

In the diagram below, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF: FD = 4: 3. E is a point on AM such that EF//BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

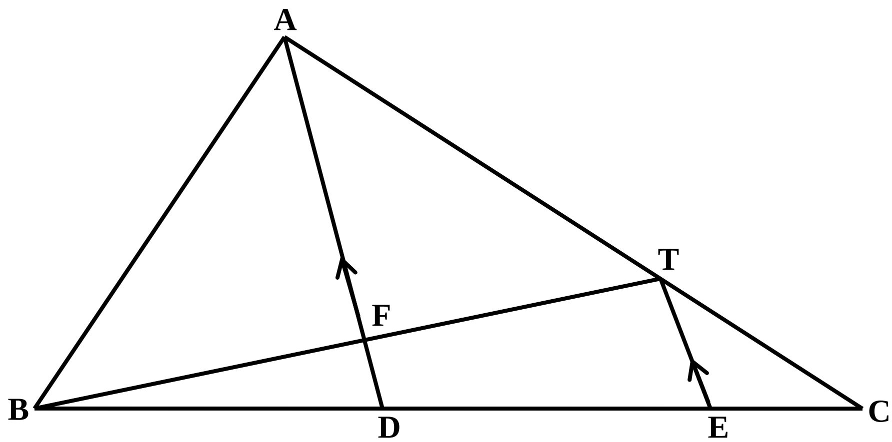
23.1 $\frac{EM}{AM}$

23.2 $\frac{CM}{ME}$

23.3 $\frac{\text{Area } \Delta \text{ FDC}}{\text{Area } \Delta \text{ BDC}}$

QUESTION 24

In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm. AT: TC = 2:1 and AD $\parallel$ TE.



24.1 Write down the numerical value of $\frac{CE}{ED}$.

24.2 Show that D is the midpoint of BE.

24.3 If FD = 2 cm, calculate the lengths of TE.

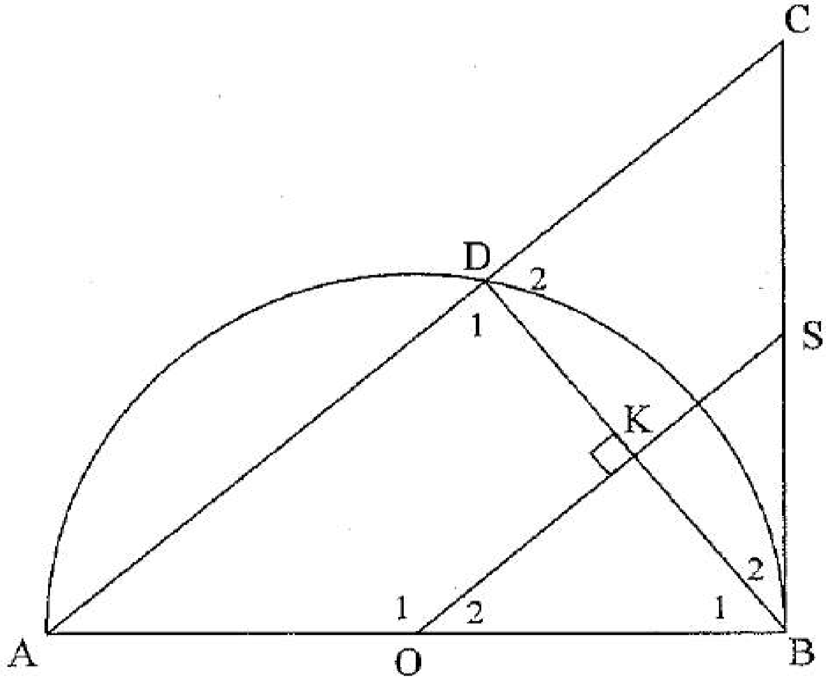
24.4 Calculate the numerical value of:

24.4.1 $\frac{Area\ of\ \Delta\ ADC}{Area\ of\ \Delta\ ABD}$

24.4.2 $\frac{Area\ of\ \Delta\ TEC}{Area\ of\ \Delta\ ABC}$

QUESTION 25

In the diagram below, AOB is the diameter of semi-circle ADB with O the centre. ADC is a straight line. CB is a tangent at B. OK $\perp$ DB with K on DB. OK produced cuts CB at S.



25.1 Prove that:

25.1.1 $OS \parallel AC$

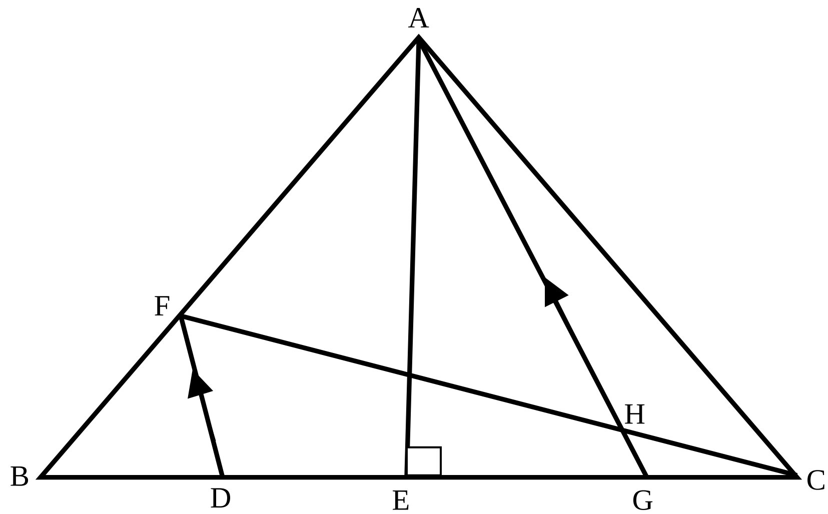
25.1.2 $\Delta ABC \,|||\, \Delta ADB \,|||\, \Delta BDC$

25.1.3 $AB^2 = AD \cdot DC$

25.2 Calculate the numerical value of $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta OSB}$

QUESTION 26

In the diagram below, $\frac{BG}{BC}=\frac{2}{3}$ and $\frac{BE}{EA}=\frac{1}{2}$. If AG // DE, determine:



26.1 EH :HC

26.2 $\frac{Area\,\Delta\,BAG}{Area\,\Delta\,CAG}$