

# **CAPE WINELANDS EDUCATION DISTRICT**

## **COMMON PAPER**

**GRADE 12**

**MATHEMATICS P1  
SEPTEMBER 2018**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 11 pages and 1 information sheet.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

**QUESTION 1**

1.1 Solve for  $x$  in each of the following:

1.1.1  $x^2 - 10x - 3 = 0$  (*correct to 2 decimal places*) (3)

1.1.2  $x(x + 2) - 9(x + 2) = 0$  (3)

1.1.3  $3^x(3^x - 9) = 0$  (2)

1.2 The roots of the equation  $y = ax^2 + bx + c$  are given as:

$$x = \frac{4 \pm \sqrt{16 - 4k}}{2}$$

1.2.1 For which values of  $k$  will the roots be real? (2)

1.2.2 Give one value of  $k$  for which the roots will be rational, if  $k \in \mathbb{Z}$  (integers) (1)

1.3 Solve for  $x$  and  $y$  simultaneously:

$2x + y - 2 = 0$  and  $x(x - y + 1) = 0$  (6)

1.4 Given:  $f(x) = \sqrt{(x - 1)(x + 2)}$  and  $g(x) = 2x + 3$

1.4.1 For which value(s) of  $x$  will  $f(x)$  be non-real? (3)

1.4.2 Solve for  $x$  if  $f(x) = g'(x)$  (5)

[25]

**QUESTION 2**

2.1 Prove that in any arithmetic series of which the first term is  $a$  and where the constant

difference is  $d$ , the sum of the first  $n$  terms is given by  $S_n = \frac{n}{2}[2a + (n - 1)d]$  (4)

2.2 Given the following sequence:  $-5 ; -1 ; 3 ; 7 ; \dots ; 35$

2.2.1 Determine the number of terms in the sequence. (3)

2.2.2 Calculate the sum of the sequence. (2)

- 2.3 For an arithmetic series consisting of 15 terms,  $S_n = 2n - n^2$   
Determine:

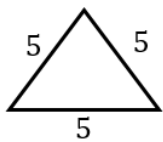
2.3.1 the first term of the sequence. (2)

2.3.2 the sum of the last 3 terms. (3)

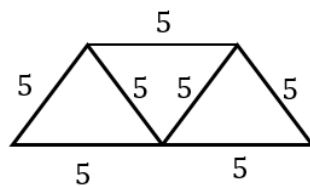
- 2.4 A series of bridges, formed by equilateral triangles, are built with steel bars with length 5 m each.

Please note: the length of the bridge (driving surface) is the length of the bottom side(s)

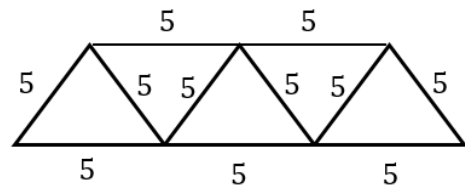
Arrangement 1



Arrangement 2



Arrangement 3



Calculate how many bars are needed for a bridge with a driving surface of 75 m.

(3)  
[17]

### QUESTION 3

- 3.1 A quadratic number pattern  $T_n = an^2 + bn + c$  has a third term equal to  $-1$ , while the first differences of the quadratic sequence are given by:  $-12; -8; -4$

3.1.1 Write down the values of the first four terms of the quadratic sequence. (2)

3.1.2 Calculate the value of  $a, b$  and  $c$ . (3)

- 3.2 Consider the geometric series  $4 + p + \frac{p^2}{4} + \frac{p^3}{16} + \dots$

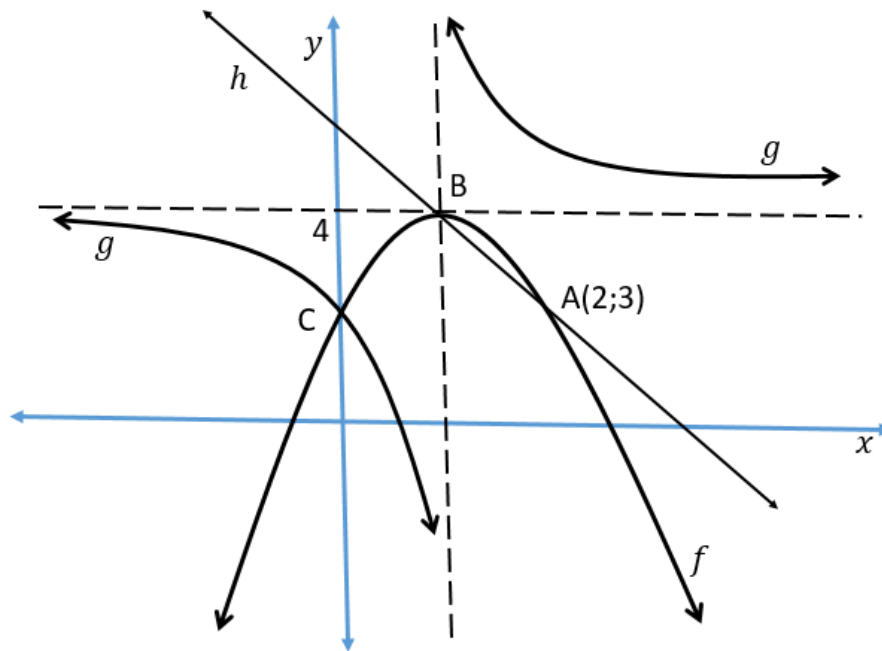
3.2.1 Calculate the value(s) of  $p$  for which the series converges. (2)

3.2.2 Calculate the value of  $p$  if the sum to infinity is 3. (3)

[10]

**QUESTION 4**

In the sketch below, the graphs of  $g(x) = \frac{k}{x+p} + 4$  and  $f(x) = ax^2 + bx + c$  are given. The asymptotes of  $g$  intersect at B, the turning point of  $f$ . The graphs of  $f$  and  $g$  intersect at C. The axis of symmetry of  $g$  that has a negative gradient, is the line  $h(x)$  that intersects the graph of  $f$  at A (2; 3) and B.

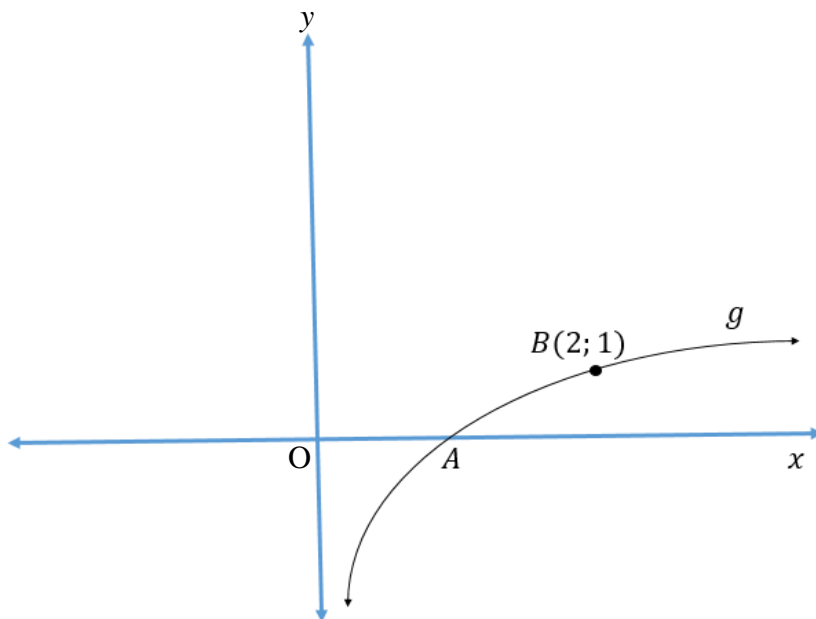


- 4.1 Determine the equation of  $h$ . (2)
- 4.2 Show that the coordinates of B are (1; 4).  
Clearly show all calculations. (2)
- 4.3 Show that the equation of  $f(x)$  is given by  $f(x) = -x^2 + 2x + 3$ .  
Clearly show all calculations. (4)
- 4.4 Determine the equation of  $g$ . (3)
- 4.5 Determine the equations of the asymptotes of  $g(x + 1)$ . (2)
- 4.6 Determine the value(s) of  $x$  for which  $g'(x) \cdot f'(x) \geq 0$ . (2)

**[15]**

**QUESTION 5**

The graph of  $g(x) = \log_2 x$  is given below. A is the point of intersection of  $g$  with the  $x$ -axis.



- 5.1 Write down the coordinates of A. (2)
- 5.2 Determine the equation of  $g^{-1}$  in the form  $y = \dots\dots\dots$  (2)
- 5.3 Sketch the graph of  $p(x) = -g(x)$ . Clearly indicate all intercept(s) with the axes as well as the coordinates of any other point on the graph. (2)
- 5.4 Use the graph to determine for which value(s) of  $x$  will  $\log_{1/2} x \geq 0$  (2)
- [8]**

**QUESTION 6**

Sketch the graph of  $f(x) = \frac{k}{x+p} + q$  if:

- the domain is given as:  $x \in R ; x \neq -1$ .
- the range is given as:  $y \in R ; y \neq 2$ .
- $k < 0$
- the  $x$  – *intercept* is:  $(-\frac{1}{2}; 0)$ .
- $f(0) = 1$

**[5]****QUESTION 7**

7.1 After how many years will the price of a vehicle that has a value of R85 000 depreciate to R48 000 if the depreciation rate is 13,4% p.a. on the reducing balance? (4)

7.2 A man takes out a mortgage loan for a house. He plans to pay back the loan monthly over a period of 20 years. The size of the bond is R600 000 and the interest rate is calculated at 11% p.a. compounded monthly.

7.2.1 Calculate his monthly repayment. (4)

7.2.2 After 14 years he decides to settle the mortgage loan. Calculate the outstanding balance after 14 years. (3)

7.2.3 The man can only pay R200 000 of the settlement amount after 14 years. He makes an arrangement with the bank to settle the rest of the loan amount 3 months later in one payment.

Calculate the value of the second settlement amount if :

- he doesn't make any further payments on the loan
- he will pay a once-off amount three months later
- the interest rate remains the same.

**(3)****[14]**

**QUESTION 8**

8.1 Given:  $f(x) = 3 - x^2$

8.1.1 Determine  $f'(x)$  from first principles. (5)

8.1.2 Determine the gradient of the tangent to  $f$  where  $x = -1$ . (2)

8.2 Differentiate with respect to  $x$ :

8.2.1  $f(x) = -3x^3 - 2\sqrt{x}$  (3)

8.2.2  $xy = \left(x - \frac{1}{x^2}\right)\left(x + \frac{1}{x^2}\right)$  (4)

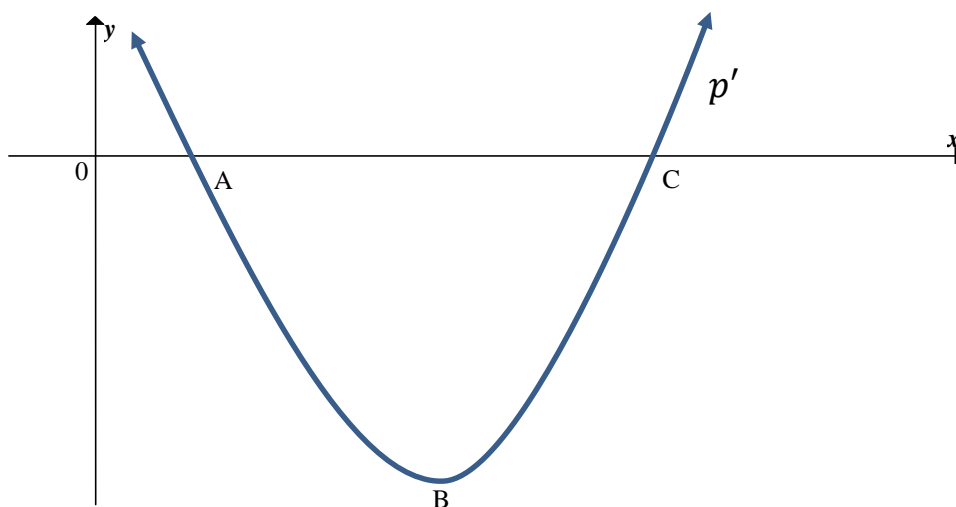
8.3 Given the functions  $h(x) = \frac{k}{x}$  and  $g(x) = 3x + 6$

8.3.1 Determine the equation of  $h'(x)$  in terms of  $k$ . (2)

8.3.2 Calculate the value of  $k$  if  $g$  is a tangent to  $h$ . (5)



- 8.4 The sketch below shows the graph of  $p'(x)$  where  $p(x) = x^3 + bx^2 + 24x + c$ .  $A(2;0)$  is an  $x$ -intercept of both  $p(x)$  and  $p'(x)$ .  $C$  is the other  $x$ -intercept of  $p'(x)$ .



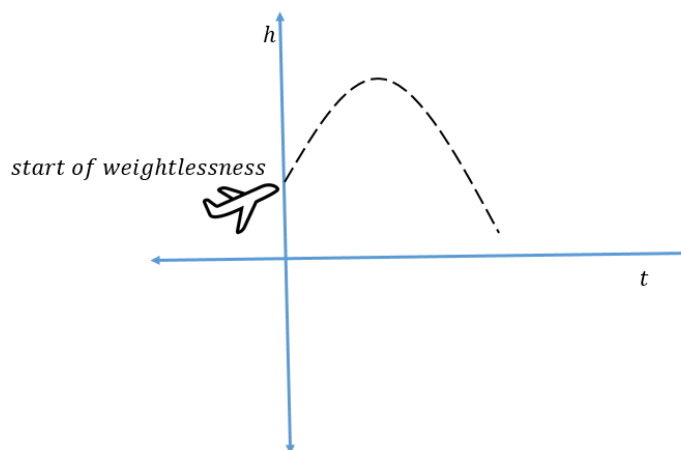
- 8.4.1 Show that the numerical value of  $b$  is equal to  $-9$ .  
Clearly show all your calculations. (3)
- 8.4.2 Calculate the coordinates of  $C$ . (3)
- 8.4.3 For which value(s) of  $x$  will  $p(x)$  be increasing? (3)
- 8.4.4 Calculate the value(s) of  $x$  for which the graph of  $p$  is concave up. (2)
- 8.4.5 Sketch a possible graph of  $p(x)$ . Clearly indicate the  $x$ -coordinates of the turning points and the point of inflection. (4)
- [36]

**QUESTION 9**

The parabolic flight of an aeroplane that uses weightlessness, can be modelled by the quadratic equation

$$h(t) = -10t^2 + 300t + 9750$$

where  $h$  is the height of the aeroplane, in metres, and  $t$  the time, in seconds, when weightlessness is reached.



- 9.1 Determine the height when weightlessness is reached initially. (2)
- 9.2 Calculate the number of seconds that it takes the aeroplane to reach its maximum height after weightlessness has been reached. (3)
- 9.3 Calculate the number of seconds (duration) of weightlessness if weightlessness is lost at the same height as when it is reached. (2)
- [7]**

**QUESTION 10**

- 10.1 In a certain survey, 23% of the people who were interviewed, said their favourite ice-cream is Vanilla.

What is the probability that the favourite ice-cream of a person, chosen randomly, is NOT vanilla flavour? (3)

- 10.2 At a certain political meeting there are 20 Republicans (R), 13 Democrats (D) and 6 members of the Independent party (I).

What is the probability that a person, chosen randomly, will be a Democrat or a Republican? (3)

- 10.3 There are 8 swimmers who will participate in a certain Olympic event. Two of the qualifying participants are from France.

In how many different ways can the swimmers be arranged if the French swimmers must swim next to each other? (2)

- 10.4 Consider the word: **CAPETOWN**

Answer the following questions, if repetition of letters are not allowed:

- 10.4.1 How many different possible arrangements can be compiled from the word CAPETOWN? (2)

- 10.4.2 Calculate the probability that the word will start with a C and end with an N. (3)

**[13]**

**TOTAL: 150**

## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{oppervlakte } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$