GANSBAAI ACADEMIA

MATHEMATICS
Grade 12

EXAM P2
September 2014
Total: 150
Time: 3 hours

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INSTRUCTIONS

1. This question paper consists of 9 questions. Answer ALL questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number your answers correctly according to the numbering system used in this question paper.
6. Diagrams are not necessarily drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.

PAPER 2

QUESTION 1

The tuck shop at Great Future High School sells cans of soft drinks. The Environmental Club at the school decided to have a can-collection project for three weeks to make learners aware of the effects of litter on the environment.

The data below shows the number of cans on each school day of the three-week project:

58	83	85	89	94
97	98	100	105	109
112	113	114	120	145

1.1 Calculate the mean of cans collected over the three-week period. (2)

1.2 Calculate the standard deviation. (2)

1.3 Determine the lower and upper quartiles of the data. (2)

1.4 Draw a box and whisker diagram to represent the data on DIAGRAM SHEET 1. (3)

1.5 On how many days did the number of cans collected lie outside ONE standard deviation of the mean? (3)

[12]

QUESTION 2

A training manager wants to know if there is a relationship between the hours spent on training (x) and a specific category of employee and their productivity (units delivered per day, y)

The following data is from the files of 10 employees.

Employees	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Hours training (x)	16	36	20	38	40	30	35	22	40	24
Productivity (units delivered per day, y)	45	70	44	56	60	48	75	60	63	38

2.1 Draw a scatter plot of the data on DIAGRAM SHEET 2. (2)

2.2 Determine the equation of the linear regression line of this data. (2)

2.3 Use your regression equation to determine the productivity level for an employee who has received 22 hours of training. (2)

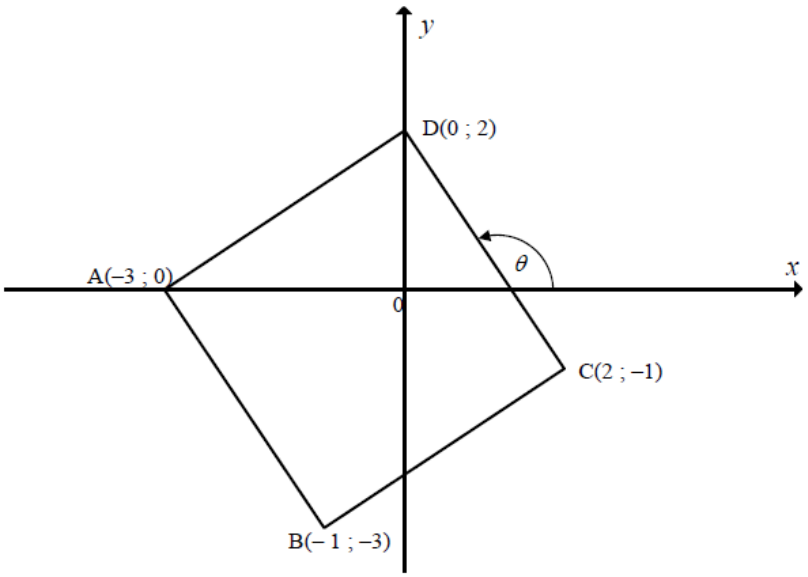
2.4 Determine the correlation between productivity and hours training. (2)

2.5 How do you interpret the correlation, comment? (2)

[10]

QUESTION 3

ABCD is a quadrilateral with vertices A(– 3 ;0), B(– 1 ; – 3), C(2 ; –1) and D (0 ; 2).



3.1 Determine the coordinates of M, the midpoint of AC. (2)

3.2 Show that AC and BC bisect each other. (3)

3.3 Prove that $A\hat{D}C = 90°$. (4)

3.4 Show that ABCD is a square. (6)

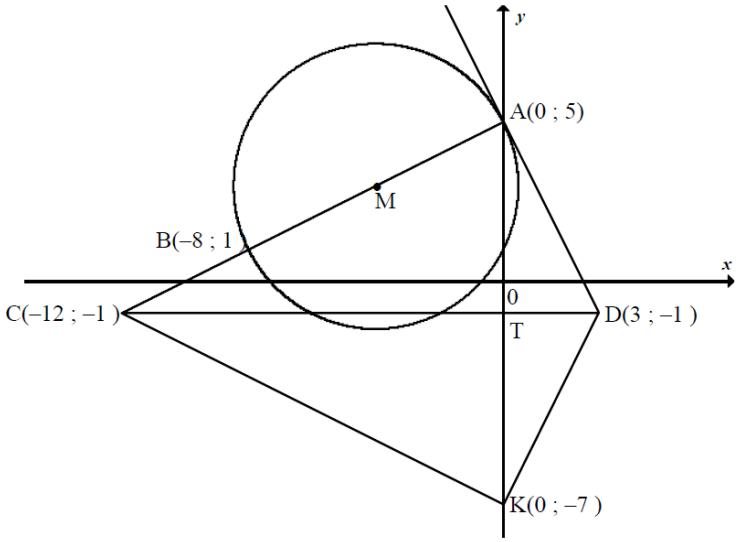
3.5 Determine the size of θ, the angle of inclination of DC, correct to ONE decimal place. (3)

3.6 Does C lie inside or outside the circle with centre (0 ; 0) and radius 2? Justify your answer. (2)

[20]

QUESTION 4

A(0 ; 5) and B(–8 ; 1) are two points on the circumference of the circle centre M, in a Cartesian Plan. M lies on AB. DA is a tangent to the circle at A. The coordinates of D are (3 ; –1) and the coordinates of C are (–12 ; –1). Points C and D are joined. K is the point (0 ; –7). CTD is a straight line.



4.1 Show that the coordinates of M, the midpoint of AB, are (– 4 ; 3). (1)

4.2 Determine the equation of the tangent AD. (4)

4.3 Determine the length of AM. (3)

4.4 Determine the equation of circle centre M in the form
$ax^2 + by^2 + cx + dy + e = 0$ (4)

4.5 Quadrilateral ACKD is one of the following:
Parallelogram; Kite; Rhombus; Rectangle
Which one is it? Justify your answer. (4)

[16]

QUESTION 5

5.1 Simplify: $\dfrac{\cos(-\theta).\tan(180°-\theta).\cos(90°-\theta)}{\sin(180°-\theta).\sin(540°+\theta)}$ (7)

5.2 Given: $3\cos A+5=0$ and $A\in[0°;180°]$. Determined WITHOUT using a calculator the values of:

5.2.1 $SinA$ (3)

5.2.2 $Sin2A$ (3)

5.3 Prove that $\tan x = \dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ (6)

5.4 Determine the general solultion for: $\sin 2x-\cos x=0$. (8)

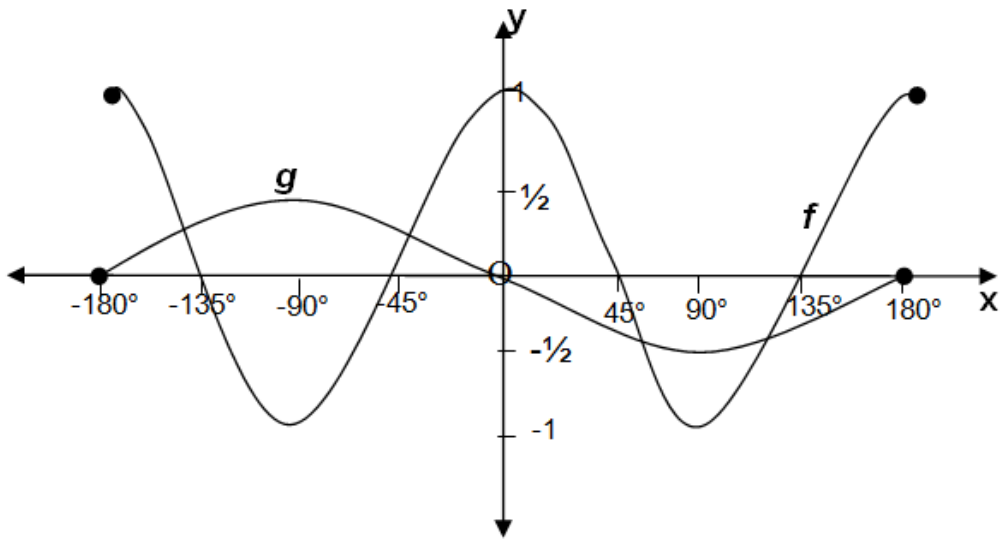
5.5 If $\cos 38°=a$, show $\dfrac{\sin 38°+\cos 52°}{\cos 52°\cos 14°+\sin 52°\sin 14°}$ in terms of a. (5)

[32]

QUESTION 6

6. The graph in the figure represents the curves of:

$f(x)=\cos ax$, -180° ≤ x ≤ 180° and $g(x)=b\sin x$, -180° ≤ x ≤ 180°



6.1 Determine a and b. (2)

6.2 Write down the period of g. (1)

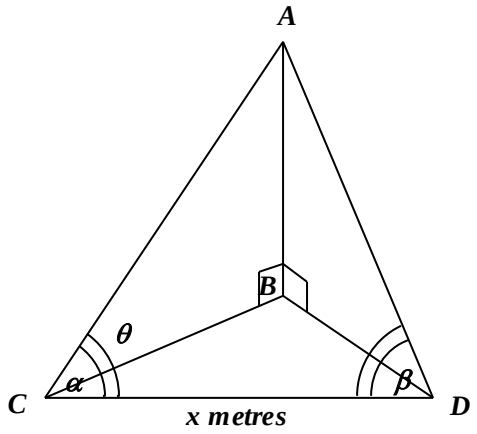
6.3 What is the minimum value of $f(x)$? (1)

6.4 For how many values of x will $f(x)+\frac{1}{2}=g(x)$ in the given interval? (2)

[6]

QUESTION 7

AB is a vertical tower in a horizontal plane BCD. The angle of elevation of A from C is θ, $A\hat{C}D=\alpha$, $A\hat{D}C=\beta$ and the distance $CD=x$ *meters*.



7.1 Prove that the height of the tower AB = $\frac{x\sin\theta\sin\beta}{\sin(\alpha+\beta)}$ (5)

7.2 Calculate the height of the tower if x = 40 *meters*, $\alpha=50^0$, $\beta=70^0$ and $\theta=15^0$ (2)

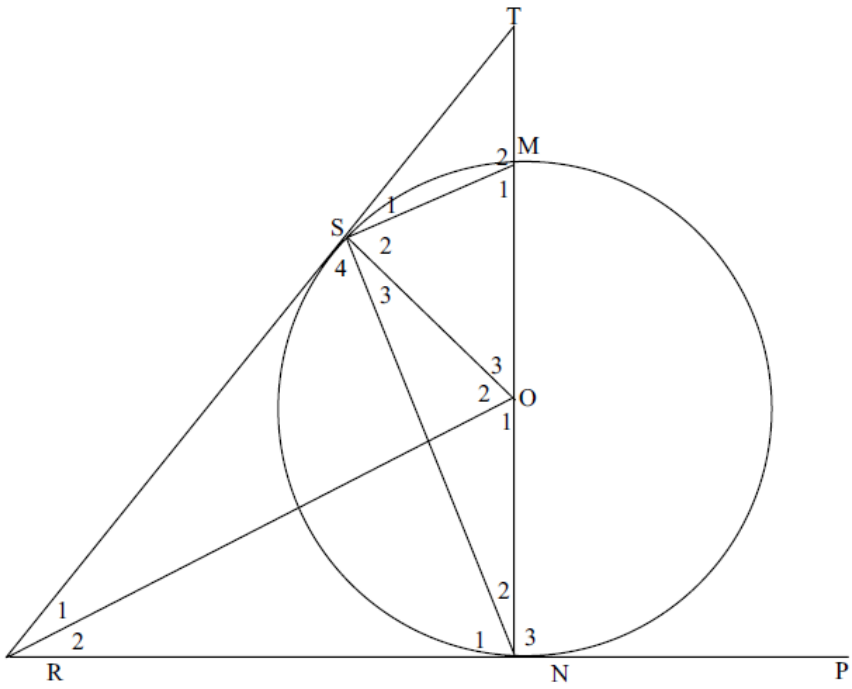
7.3 Calculate the area of ΔACD. (3)

[10]

QUESTION 8

8.1 Complete the following theorem statement:
Opposite angles of a cyclic quadrilateral are … (1)

8.2 In the figure alongside, RS and RNP are tangents to the circle with centre O at the points S and N. Radius NO is produced and cuts the circle at M and meets RS produced at T.



8.2.1 Why is $O\hat{S}R = 90°$? (1)

8.2.2 Prove that RNOS is a cyclic quadrilateral. (4)

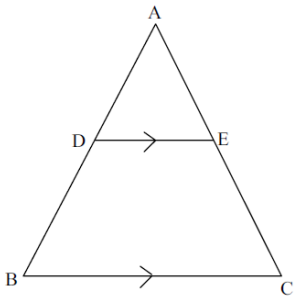
8.2.3 If $\hat{S}_1 = x$, determine, with reasons, FOUR other angles in the figure which are equal to x. (8)

8.2.4 Prove that: $\hat{S}_3 = \frac{1}{2}\hat{O}_3$. (4)

[18]

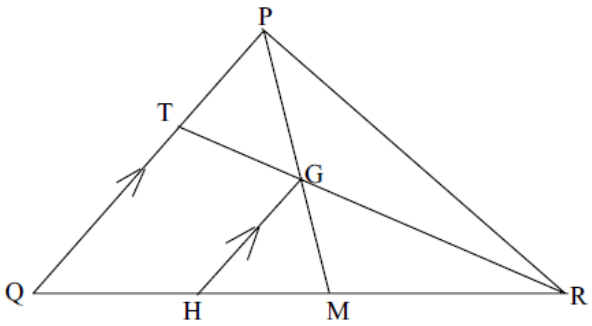
QUESTION 9

9.1 Given Δ ABC with DE // BC as shown in the figure below:



Prove that: $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

9.2 In the diagram below, M is the midpoint of QR in Δ PQR. T is a point on PQ such that PM and TR intersect at G. GH // PQ with H on QR. PG : PM = 1 : 3.

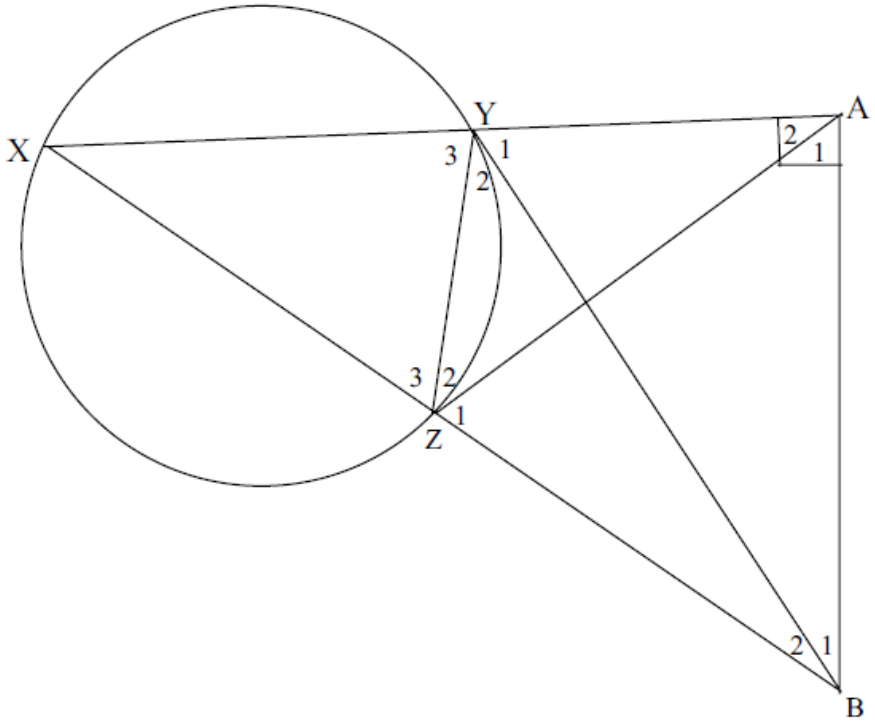


Determine, with reasons, the numerical value of:

9.2.1 $\frac{QH}{HM}$ (3)

9.2.2 $\frac{RG}{RT}$ (5)

9.3 In the diagram below, the chord XY is produced by its own length to A, and tangent AZ is drawn to touch the circle at Z. XZ is produced to meet the line perpendicular to XA at B and AZ = AB.



9.3.1 Prove that $\hat{Z}_3 = 90°$ (5)

9.3.2 Prove that ΔAXZ // ΔAZY. (3)

9.3.3 Prove that $AZ^2 = AY.\ XA$. (1)

[23]

TOTAL 150

Information sheet: Mathematics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad A = P(1+i.n) \qquad A = P(1+i)^n$$

$$A = P(1-i)^n \qquad A = P(1-i.n)$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r-1} \;\; ; \;\; r \neq 1 \qquad \sum_{i=1}^{n} ar^{i-1} = \frac{a}{r-1} \;\; ; \;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right) \qquad y = mx + c$$

$$y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta \qquad (x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC\ ; \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \qquad \cos 2\alpha = 1 - 2\sin^2\alpha \qquad \cos 2\alpha = 2\cos^2\alpha - 1$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \bar{x} = \frac{\sum fx}{n} \qquad \text{var} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$$SD = \sqrt{\frac{\sum_{i=1}^{n} (x - \bar{x})^2}{n}} \qquad P(A) = \frac{n(A)}{n(s)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

DIAGRAM SHEET 1

QUESTION 1.4

NAME AND SURNAME: ……

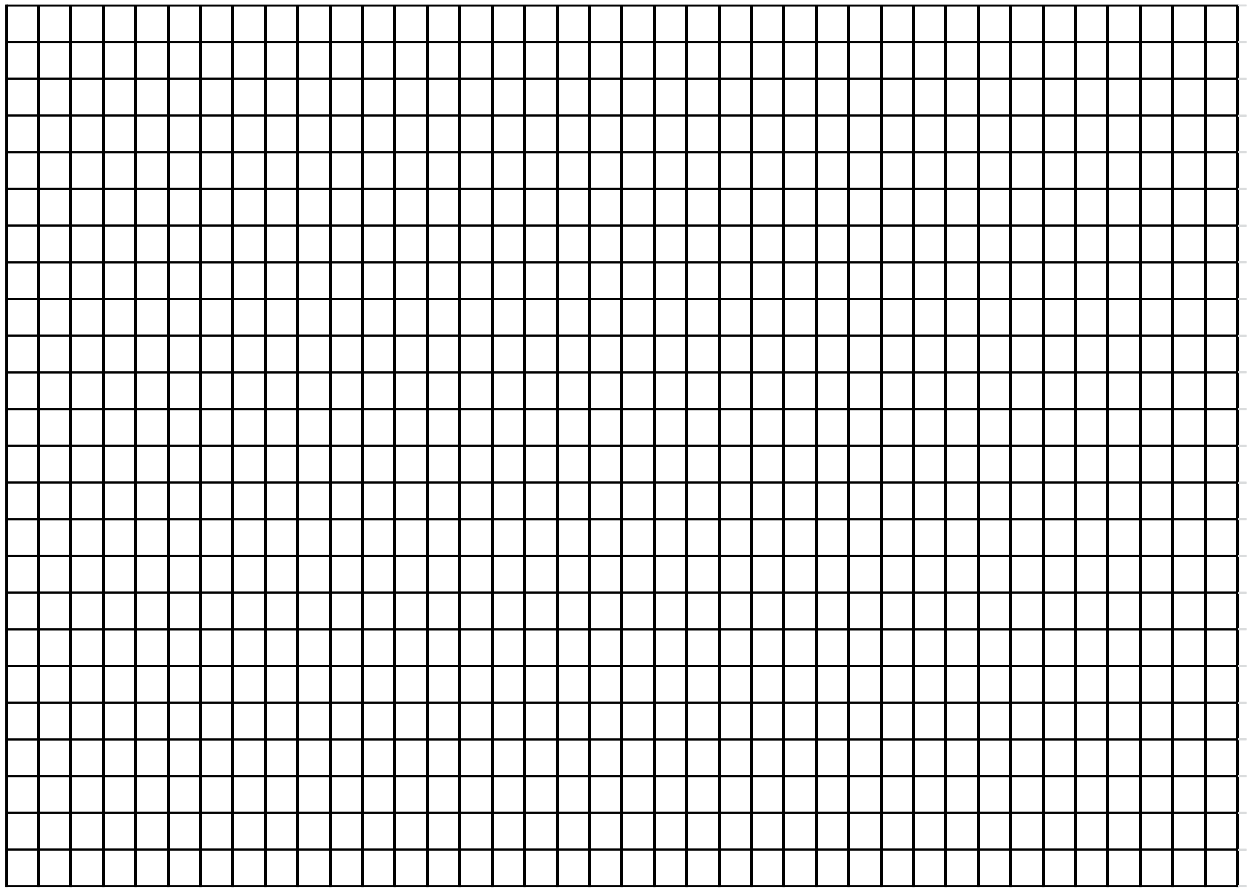


DIAGRAM SHEET 2

QUESTION 2.1

NAME AND SURNAME: …….