GANSBAAI ACADEMIA

MATHEMATICS
Grade 12

EXAM P1
September 2014
Total: 150
Time: 3 hours

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INSTRUCTIONS

1. This question paper consists of 11 questions. Answer ALL questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number your answers correctly according to the numbering system used in this question paper.
6. Diagrams are not necessarily drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.

PAPER 1

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 11x = -24$ (3)

1.1.2 $6^{3x-1} = 1$ (3)

1.1.3 $\sqrt{7x+2} + 2x = 0$ (5)

1.2 Determine the values for x for which $\sqrt{x(x+4)+3}$ will be a real number. (5)

1.3 Solve x and y simultaneously for the following equations:

$4y + 3x = 50$ and $x^2 + y^2 = 100$ (7)

1.4 Simplify, without using a calculator:

$\left(\sqrt{\sqrt{13}+2}\right)\left(\sqrt{\sqrt{13}-2}\right)$ (3)

[26]

QUESTION 2

2.1 Given the arithmetic sequence: $-10;\ -3;\ 4;\ 11$.

Determine:

2.1.1 The value of T_{50}. (2)

2.1.2 How many terms should be added to give a total of 3152. (4)

2.2 The second term of a geometric series is 24 and the eight term is 0,375.

2.2.1 Determine the common ratio and the first term. (5)

2.2.2 Determine the sum to infinity of the series. (3)

2.3 For which values of x will the series $(3x-4)^3+(3x-4)^4+(3x-4)^5+...$ be a converging series? (3)

2.4 Solve for *n* if:

$\sum_{k=1}^{n} 4.3^{n-1} = 2186$ (5)

[22]

QUESTION 3

3.1 The first differences of a quadratic sequence is $3;\ x-5;\ 23-x$. The second difference is *4*.

3.1.1 Calculate the value of x. (3)

3.1.2 Determine the first four terms of the quadratic sequence if the second term of the sequence is equal to $3x-31$. (3)

[6]

QUESTION 4

4.1 Thuso is a young farmer. He has just bought his first tractor for R500 650. Due to inflation, the value of the tractor depreciates at a rate of 7% p.a. on a reducing balance. Thuso knows that he will have to replace the tractor in four years' time. The price of a new tractor appreciates at 9% per annum.

4.1.1 Calculate the scrap value of his tractor after four years. (3)

4.1.2 Determine the cost of a new tractor in four years' time. (3)

4.1.3 He plans to trade in the old tractor after four years. In his budget, he makes provision for R50 000 unforseen expenses that might occur during the transaction. How much money will he need in the sinking fund in four years' time? (3)

4.1.4 Thuso immediately starts to pay equal monthly payments into the sinking fund. The fund earns interest at 9% per annum, calculated monthly. His last payment is made at the end of the four-year period. How much does he pay every month? (3)

4.2 Sam invests an amount of money at an interest rate of 11% per annum, compounded monthly. For how many years, correct to three decimal places, must he invest the money if he wants the amount to at least triple in value? (3)

[15]

QUESTION

Consider the function $f(x)=\frac{4}{x-1}+3$.

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the coordinates of the intercepts of the graph of f with the axes. (4)

5.3 Sketch the graph of f, on DIAGRAMSHEET 1. Clearly indicate the coordinates of the intercepts with the axes as well as the asymptotes. (4)

5.4 Calculate the average gradient of the curve between $x=4$ and $x=6$. (3)

5.5 For which values of x is $f(x)\leq 0$? (2)

[15]

QUESTION 6

Given: $f(x)=3^x$

6.1 Determine an equation for f^{-1} in the form $f^{-1}(x)=\dots$ (1)

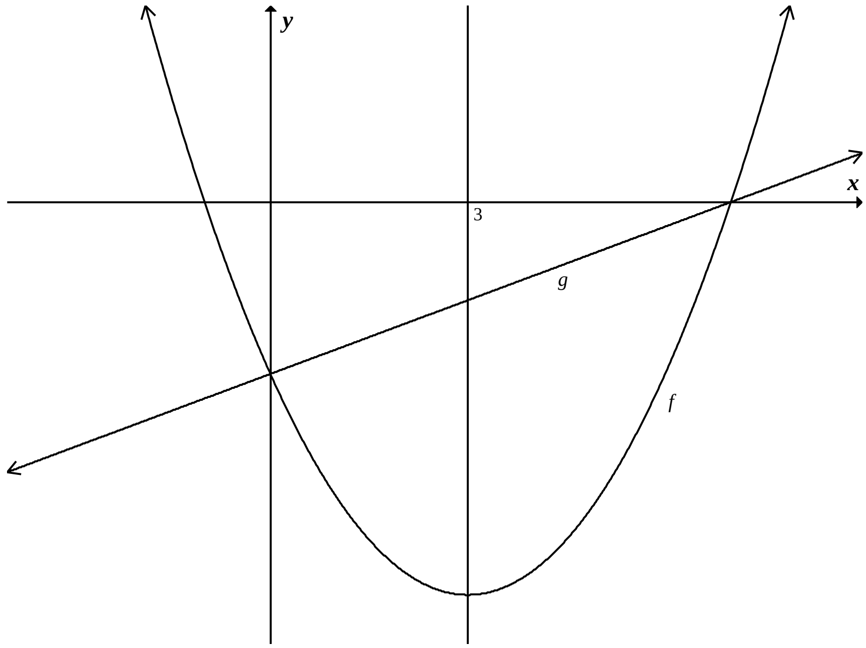
6.2 Sketch, on DIAGRAM SHEET 2, the graphs of f and f^{-1}, clearly show ALL intercepts with the axes. (4)

6.3 For which values of x will $f(x).f^{-1}(x)\leq 0$? (2)

[7]

QUESTION 7

A Parabola f intercepts the x-axes at B and C and the y-axes at E. The equation of axes of symmetry of the parabola is $x = 3$. The straight line through E and C's equation is $g(x) = \frac{x}{2} - \frac{7}{2}$.



7.1 Show that the coordinates of $C(7;0)$. (2)

7.2 Determine the x-coordinate of B. (2)

7.3 Determine the equation of f in the form $y = a(x-p)^2 + q$. (6)

7.4 Write down the equation of the graph h, if it is the reflection of f in the x-axes. (1)

[11]

QUESTION 8

8.1 Determine $f'(x)$ from first principle if $f(x) = 2x^3$. (5)

8.2 Evaluate (Leave your answers with positive exponents where applicable):

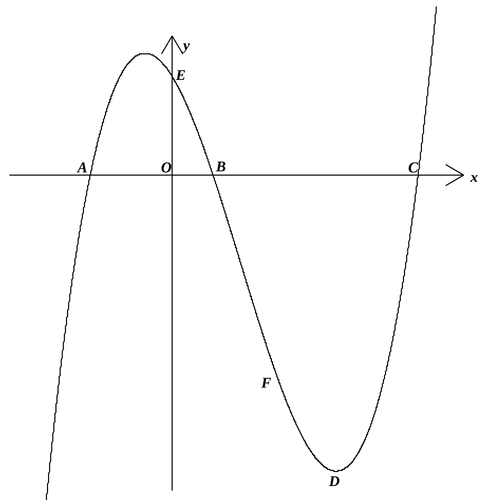
8.2.1 $\frac{dy}{dx}$ if $y = \frac{x^2 - 5x + 6}{x - 3}$ (3)

8.2.2 $D_x\left[\frac{4}{\sqrt{x}} - \frac{x^3}{9}\right]$ (4)

[12]

QUESTION 9

Sketched below the graph $f(x)=x^3-5x^2-8x+12$.



Determine:

9.1 The coordinates of A, B and C. (5)

9.2 The coordinates of the local minimum D. (4)

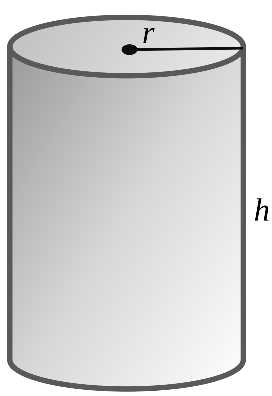
9.3 The coordinates of the point of inflection on the curve between B and D. (4)

9.4 The equation of the tangent to the curve at the point E where $y = f(x)$ cuts the y -axes. (3)

[16]

QUESTION 10

A cool drink factory wants to minimize the amount of aluminium they use to make one can. A cool drink can has a capacity of 340ml. Allow a further 22cm^2 for seams and so forth.



10.1 Determine the height, h, in terms of x. (2)

10.2 Determine an expression for the outer surface area in terms of x and π. (3)

10.3 Determine a value of x that will minimize the amount of aluminium per can. (4)

[9]

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

P(B) = 2P(A)
P(A or B) = 0,57

Calculate P(B). (3)

11.2 The letters of the word "S K R E E U E R S" are arranged randomly to form a new word.

11.2.1 How many new words can be made? (3)

11.2.2 How many words will start and end with a R? (3)

11.2.3 What is the probability that a word chosen randomly, will start and end with the letter R? (2)

[11]

TOTAL 150

Information sheet: Mathematics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + i.n) \qquad A = P(1+i)^n$$

$$A = P(1-i)^n \qquad A = P(1 - i.n)$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r-1} \ ; \quad r \neq 1 \qquad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{r-1} \ ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \ ; \ \frac{y_1 + y_2}{2}\right) \qquad y = mx + c$$

$$y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta \qquad (x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC \ ; \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \qquad \cos 2\alpha = 1 - 2\sin^2\alpha \qquad \cos 2\alpha = 2\cos^2\alpha - 1$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \bar{x} = \frac{\sum fx}{n} \qquad \text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(x - \bar{x})^2}{n}} \qquad P(A) = \frac{n(A)}{n(s)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

DIAGRAM SHEET 1

QUESTION 5.3

NAME AND SURNAME: ……………………………………………………………………………………

DIAGRAM SHEET 2

QUESTION 6.2

NAME AND SURNAME: …………………………………………………………………………………….